Bus Conductors' Use of Mental Computation in Everyday Settings – Is it Their Ethnomathematics?

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Abstract

Although over the past 15 years, mathematics education research has begun to explore the nature of the mathematics used in different workplaces, the research field of workplace mathematics is still in its infancy. Guided by the desire to add to the mathematics education research in India, the general aim of this study is to develop a better understanding of the mathematics used in everyday situations. To this end, I focused on the workplace mathematics of bus conductors in Chennai, India. More specifically, the purpose of this study is to observe, understand, analyze, and describe the mental mathematical practices of the bus conductors in their workplace and examine what this knowledge can add to the study of everyday mathematics.

Introduction

Mathematics knowledge has been traditionally considered as absolute and infallible (Ernest, 1991). This absolutist view of mathematics has been challenged since the beginning of the twentieth century and a new wave of *fallibilist* philosophies of mathematics has evolved. Researchers have recognized that the study of mathematical knowledge has a value dimension and hence it is a social and a cultural phenomenon as well (Bishop, 1988). This evolution has greatly influenced mathematics education research in the past few decades and has resulted in the rise of different areas of research that include ethnomathematics, everyday mathematics, situated cognition, and workplace mathematics (Presmeg, 2007). The essential principle guiding this line

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of research is the acknowledgement of the fact that people in several walks of life perform mathematical activities out of school, at home, and at work.

Ubiratan D'Ambrosio, widely known as the *father of ethnomathematics* initiated the ethnomathematical program in the 1970s as a methodology to "track and analyze the processes of generation, transmission, diffusion and institutionalization of mathematical knowledge in diverse cultural systems" (D'Ambrosio, 1990, p. 78). Vithal and Skovsmose (1997) added a new dimension to the term ethnomathematics by stating, "ethnomathematics can refer to a certain practice as well as the study of this practice" (p. 133). Although the term ethnomathematics has been interpreted by researchers in more than one way, they are guided by the belief that the role of culture is significant in the learning and teaching of mathematics.

One area of research that emanates from the research field of ethnomathematics is the research area of everyday mathematics (Vithal & Skovsmose, 1997). Moschkovich (2002) coined the term *everyday mathematics* to denote the "mathematical practices that adults or children engage in, other than school or academic mathematics" (p. 2) and I use it here to denote the same. One area of study that has stemmed from the research field of *everyday mathematics* is concerned with investigating the mathematical practices of adults in various workplaces. This line of research gives some insight into how people conceptualize the role of mathematics in their work.

Although the past two decades have seen a surge in research dealing with the workplace mathematics of adults, many of these studies were conducted in the western hemisphere and developed nations of the world. Very few studies have investigated the nature of workplace mathematics in developing nations (e.g., Mukhopadhyay, Resnick, & Schauble, 1990). The research reported in this paper is part of a larger project that addresses this lacuna by investigating the workplace mathematics of bus conductors in Chennai, India. In particular, the research purpose associated with this study was to observe, understand, describe, and analyze the mental mathematical practices of bus conductors in their workplace and examine what this knowledge can add to the study of everyday mathematics and hence ethnomathematics.

In this paper, I specifically examine one bus conductor's use of mental computation as he engaged in work-related activities; characterise his workplace mathematics using Saxe's (1991) four parameter model and thereby claim that his workplace mathematics is indeed an example of ethnomathematics.

Literature Review

Starting with a broad theoretical field of ethnomathematics, I narrowed my focus to concentrate workplace mathematics, which could be viewed as a subset of the research field of everyday mathematics. In this section, I will provide a brief overview of research related to ethnomathematics, everyday mathematics and workplace mathematics.

Vithal and Skovsmose (1997) classified research on ethnomathematics into four strands. The first strand is concerned with challenging the traditionally told history of mathematics, the second strand is related to analyses of mathematics associated with the traditional cultures of indigenous people; the third strand explores the mathematics of different groups of people in everyday settings, and the fourth strand focuses on the relationship between ethnomathematics and mathematics education.

Strand One

Mathematical thinking and learning take place in any culture and it is important to study the mathematics of different groups of people from all over the world (Ascher, 2002). The interaction between different cultures of the world has contributed to the growth of mathematics.

It is also true that much of the so-called Western mathematics "originated in the ad hoc practices and solutions to problems developed by small groups in particular societies" (Katz, 2003, p. 557). In spite of this fact, traditionally told histories of mathematics have neglected the contributions to mathematics from the non-European cultures and have presented a "Eurocentric view" of mathematics (Joseph, 1987, p.16). Contributions to this strand of ethnomathematics assume significance because they counter Eurocentrism of the history of mathematics by providing alternate perspectives, paying particular attention to the historical development of cultures (Katz).

Strand Two

Research belonging to this strand of ethnomathematics has revealed the diversity of mathematical systems and practices in different cultures around the world (Ascher, 1991). This line of research includes works of anthropologists who have investigated the everyday mathematical practices of people belonging to different cultures all over the world. This body of research includes investigation of mathematical ideas existing in non-Western societies. Since the launch of the ethnomathematical movement in the 1980s, several researchers have carried out investigations that discuss the mathematics of specific cultural groups in Africa, Australia, and the Americas (e.g., Gerdes, 1996; Saxe, 1991).

Strand Three

This strand of ethnomathematics is specifically connected to the mathematics used by people in everyday settings. This line of research has investigated the thinking and practices of participants in situations where they developed mathematical knowledge in a social context (e.g., Jurdak & Shahin, 1999; Guberman, 1996; Carraher, Carraher, & Schliemann, 1987). Researchers have employed ethnographical methods to describe mathematical ideas embedded in the everyday practices of children and adults.

One area of study that has stemmed from the research field of everyday cognition is concerned with investigating the mathematical practices of adults in various workplaces. Some of the studies that have contributed to the literature in this field include Millroy's (1992) study that investigated the mathematics involved in carpenters' workplaces, Masingila's (1994) investigations of the mathematical practices of carpet layers, and Noss, Hoyles and Pozzi (2000) explorations of the mathematics involved in the work of bank employees, nurses, and commercial pilots. The findings from all of these studies reiterate that adults reason in problematic situations quite differently from the way mathematicians do and "problem solving at work is characterized by a pragmatic agenda and geared to solving particular problems" (Noss, Hoyles & Pozzi, p. 17).

Strand Four

`The fourth strand of research on ethnomathematics, is interconnected to the other three strands and is of particular interest to a mathematics educator. This strand specifically aims to address the following questions: What are the implications of ethnomathematics for the theories and practices of the mathematics education research community? How can we use this research to bridge the gap that exists between everyday and academic mathematics? How can we use this research base effectively to make mathematics appealing to students of diverse cultural groups in a classroom setting? Research studies in recent years have attempted to focus on such questions and find ways to bring components of everyday mathematical practices into the classrooms (e.g. Hall, 2000).

Research reported in this paper could be placed under the third strand of research on ethnomathematics – everyday mathematics (in particular to workplace mathematics) and is unique in the sense that it extends research on workplace mathematics into a new context and characters – Bus conductors in Chennai, India.

Theoretical Framework

The assumptions underlying the present study are that the bus conductors' workplace mathematical practices are influenced by their working conditions and that their practice-linked goals emerge and change as individual conductors participate in this practice. Hence, I needed a theoretical framework that acknowledged the influence of context on their mathematical practices. Second, I needed a framework that could inform my investigation of the cognitive processes in the social context and one that would take into consideration the complex relationships between the conductors' mathematical tasks and the practices in which they were performed.

I chose Saxe's (1991) four-parameter model (see Figure 1) to explore the research question. Saxe formulated this model in his quest for developing a framework that would account for the developmental cognitive processes of participants in a cultural practice, and that would not ignore the socio-cultural context in which the practice took place. Saxe used his model to explain the complex relationships between Oksapmin traders' goals and actions and the trading practice in which their actions were performed. Saxe's (1991) four-parameter model brings out the context related parameters that influence a bus conductor's work related goals and the mathematical goals related to his work activities.

In this paper, I will use Saxe's four parameter model to analyze and characterize one bus conductor's use of mental computation as he engaged in mental computational activities related to work.

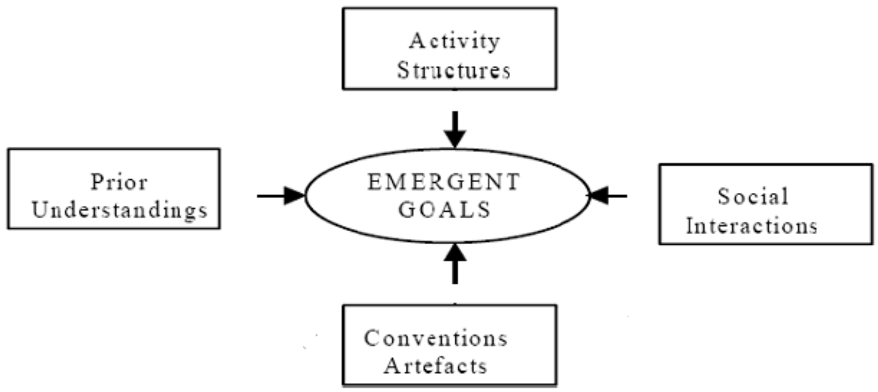

Figure 1. Four-parameter model (Saxe, 1991, p. 17)

Research Methods

The overall research study was qualitative in nature. An instrumental case study approach was employed to carry out this study. The bus conductors are employees of the government organization, Metropolitan Transport Corporation (MTC). Due to the hierarchical structures of the MTC, the higher authorities of the MTC were first contacted and their permission was sought and obtained to carry out research with the employees (bus conductors) of the MTC at their workplace. After gaining entry into the organization, a convenience sampling method was used to choose a bus depot for investigation. Once a bus depot was chosen, a purposive sampling was employed and five participants were carefully and appropriately chosen based on participants' years of service, educational qualifications, service records, and their willingness to participate in the study. The five conductors who participated in this study are called Mr. Alpha, Mr. Beta, Mr. Gamma, Mr. Delta, and Mrs. Omega (all pseudonyms). In this paper, I discuss and analyze data pertaining to one conductor – Mr. Alpha.

Data collected included official documents, field notes, summary of observations and informal conversations, transcriptions of formal and semi structured interviews, and personal reflections. Documents that were examined and used for this study included a conductor' manual, route information manual, conductors' service records, administrative documents, newspaper articles, and web sources.

Over a period of three months, I accompanied and observed Mr. Alpha during his work shifts four-six times, before, after, and during their bus trips. Observation sessions were always accompanied by short informal and semi structured interviews, which took place whenever an opportunity arose or at break times. Informal interviews were aimed at obtaining information about Mr. Alpha's perceptions of mathematics and views about the role of mathematics in his work-related activities. Semi structured interview sessions dealt with questions related to the on-site observations and his work-related documents.

Data thus collected was analysed in three stages. Stage one of the data analysis took place simultaneously as the data was collected. Stage two of data analysis moved from concrete description of observable data to a more abstract level of analysis using specific categories and themes and resulted in the creation of a case study database. This database consisted of documents, notes, narratives, tabular materials, data arrays, and so on. For the third stage of data analysis, I used Saxe's framework for in depth analysis of the data from the case study database.

Analysis and Results

In this section, first, I provide an overview of Mr. Alpha's duties at work. Next I provide brief descriptions of work-related terms (that will henceforth be used in this paper). Next, I present data that demonstrates Mr. Alpha's use of mental computation related to his work-related activities. To conclude this section, I will use a snippet identified from one of my field visits with Mr. Alpha to exemplify the use of Saxe's four –parameter model.

Mr. Alpha's Duties at Work

Every day, Mr. Alpha reported to work with several work-related goals in mind. As he described them, these goals were to have an incident free day, to avoid confrontations with the commuters, and to earn maximum collection. In addition, he set new goals that specifically related to his duties in the bus. These included issuing a ticket to a passenger and updating ticket information into an official record.

During his bus trips, Mr. Alpha commuted several times from point A to point B, picked up and dropped off commuters en route, and regulated their entry into and exit from the bus. His duties included issuing a ticket to a commuter based on the entry and exit point, tendering the exact change back to the commuter when the commuter gave more money than the required amount, and keeping a record of the number of tickets sold.

Brief Description of Terms

MTC Fleet. The MTC fleet is classified based on the number of stops buses make between the arrival and the destination points. There are ordinary, limited stop (LSS), express, deluxe, night service, vestibule, and women's special buses. Mr. Alpha was always posted on ordinary, limited service, or express buses only.

Bus Stop. These refer to intermediate stops a bus makes between the arrival and destination points to pick up and drop off passengers.

Fare Stage. A fare stage denotes a major bus stop at which fares are subject to change. There could be several bus stops between fare stages.

Ticket. This is a receipt issued by a conductor to a passenger upon receiving the fare. MTC tickets are printed in different colors for different denominations. In Figure 2, we can find a picture of a Rs. 2.50 ticket.

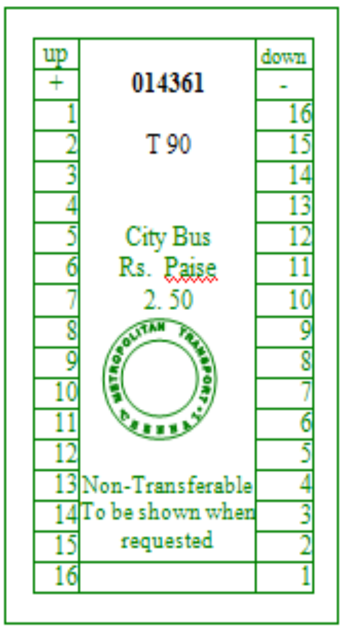

Figure 2. Rs. 2. 50 denomination city bus ticket.

Ticket Fares. Tickets fares are predetermined by the MTC based on the number of fare stages along each route and the type of service operated. Fare denotes a predetermined amount a passenger is required to pay to travel from point A to point B on a certain bus along a certain route.

Fare Stage Information. Each bus route from arrival point A to destination point B is divided into a certain number of bus stops. As mentioned earlier, some of these bus stops also serve as fare stages. In Figure 3, I have provided information regarding stages and fares for an ordinary bus.

Stage Number	Fares
1	2.00
2	2.00
3	3.00
4	3.00
5	3.50
6	4.00
7	4.00
8	4.50
9	4.50
10	5.00
11	5.00
12	5.50
13	6.00
14	6.00
15	6.00
16	6.00
17	6.00
18	6.50
19	6.50
20	6.50
21	7.00
22	7.00
23	7.00

Figure 3. Stage fare information for an ordinary bus (Value in Rupees)

For each passenger, Mr. Alpha gathered information regarding the entry point and exit point. He used this information in conjunction with the stage fare information to determine the ticket for that passenger. During one of our semi structured interviews, I requested Mr. Alpha to describe this process. Below, I present a summary of Mr. Alpha's elaborate explanation.

Each stage is approximately two kilometers long. There are at least 23 bus stops along this bus route. For an ordinary bus, the minimum fare along this bus route is Rs. 2.00 and the maximum fare is Rs. 7.00. Each column gives the schedule of ticket fares for a commuter who boards the bus at that stage. The first row gives fare information for passengers who board the bus at point

A (also fare stage 1). The second row gives fare information for commuters who board the bus at any of the bus stops after fare stage one including fare stage two, and so on. According to the current ticket pricing, a commuter traveling in an ordinary bus would pay Rs. 2.00 for the first two stages of his travel, Rs. 3.00 for the next 2 stages of his travel, Rs. 3.50 for the next stage of his travel and so on regardless of where in the route their journey starts.

The ticket fares that commuters pay for traveling in buses depend upon the fare stages where they enter and exit the bus. If commuters enter or exit the bus at any of the bus stops between two fare stages, fare is determined for the stages just before their entry points and just after their exit points. Consider a passenger who boarded an ordinary bus at a bus stop between stages five and six and exited the bus at a bus stop between stages 11 and 12. The commuter's ticket fare will be determined for stages 5 and 12. There are eight stages to be accounted for including stages 5 and 12. Looking at the second column in Table 4, we infer that the ticket fare (along stage number 8) for this commuter as Rs. 4.50.

Mr. Alpha noted that similar stage fare charts exist for the LSS, express, and deluxe buses. The starting price and the increment schedule vary for different types of buses. For an LSS bus, the starting fares at stage one will be Rs. 2.50 instead of Rs. 2.00. For an express bus, the starting fare at stage one would be Rs. 3.50 and for an express bus the starting fare would be Rs. 5.00.

To carry out a ticket transaction, Mr. Alpha first determined the number of fare stages between the entry point and the exit point, calculated ticket fares for single and multiple passengers, and determined the balance amount due to passengers, if any – Mr. Alpha completed all of these processes quickly, mentally.

Mr. Alpha's Use of Mental Computation at Work

At work, Mr. Alpha used mental computation (a) to complete ticket transactions, (b) to determine the overall ticket sales amount and (c) to determine his everyday allowance. In this paper, I will discuss in depth Mr. Alpha's mental computational activities related to phase (a).

Execution of ticket transactions. A ticket transaction process is initiated either by the conductor or by the passenger. In order to determine the ticket fare, Mr. Alpha needs the following information – passenger's entry point, exit point, and number of tickets. Some passengers, when they requested tickets, voluntarily provided this information and the correct ticket fare. When passengers did not provide the required information, Mr. Alpha sought this information from the passengers. I call the amount that passengers initially gave the conductors, the passenger paid fare (PPF). Using information obtained from passengers and the fare calculation methods appropriate to his bus types (ordinary/LSS/express), Mr. Alpha determined the actual ticket fare (ATF). He then determined if the PPF is equal to, more than, or less than the ATF. If the PPF is equal to the ATF, Mr. Alpha issued tickets and moved on to the next passenger. If the PPF is less than the ATF, he requested additional amount from the passengers. If the PPF is more than the ATF, he issued tickets and the balance amount (BA) back to the passengers.

When passengers' points of exit did not fall along his bus route, Mr. Alpha suggested alternate points of exit, mostly bus stops closest to their exit points. On most occasions, passengers accepted Mr. Alpha's suggestions. Occasionally, when a passenger rejected the conductor's suggestion, that passenger opted to get out of the bus and the conductor obliged. In such instances ticket transactions did not take place.

Based on my on-site observations and my interviews with Mr. Alpha, I identified several possible scenarios associated with ticket transactions. This information is presented in Figure 4.

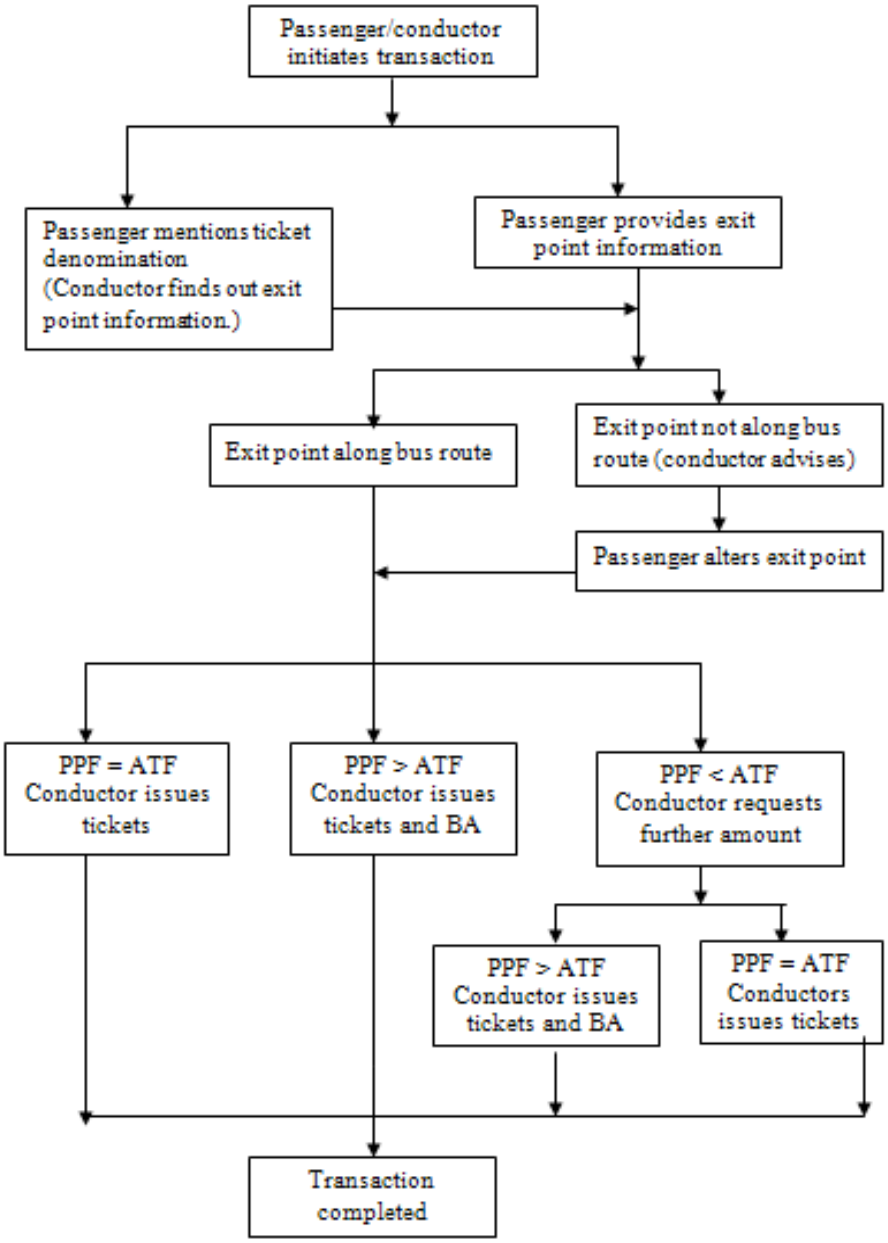

Figure 4. Possible ticket transaction scenarios.

Ticket transactions scenarios pertain to single as well as multiple passengers. The PPF could be equal to, more than, or less than the ATF. Depending on the PPF, tickets transactions could be classified into 4 categories. (a) Single passenger transactions with no balance due, (b)

Multiple passenger transactions with no balance due, (c) Single passenger transactions with balance due, and (d) Multiple passenger transactions with balance due.

I will use a snippet identified from one of my field visits with Mr. Alpha, to describe one situation depicted in Figure 4. In this situation, the passenger requested tickets by providing the exit point information. Passenger's exit point was along the bus route. The PPF was not equal to the ATF.

Snippet (Field note)

A passenger boarded the bus at point P along with four members of his family. Mr. Alpha approached the passenger. The passenger handed out a 50-rupee note and requested the conductor for five tickets to point Q. Mr. Alpha requested the passenger to give him an extra 50 paise coin and the passenger obliged. He then gave the passenger five tickets and handed 33 rupees back to the passenger.

In this situation, *both entry and exit points were fare stages*. Further, this situation involved a *multiple passenger transaction with balance due*. Mr. Alpha completed this ticket transaction in about twelve seconds. It must be noted here that regardless of the type of ticket transaction situation encountered, Mr. Alpha quickly, mentally processed relevant information and completed the ticket transaction within seconds and moved on to the next passenger.

Use of Saxe's Four-parameter Model

While Mr. Alpha described the goal associated with a ticket transaction in terms of his work-related activity, I described the goal associated with this ticket transaction as a mental mathematical activity. For example in the ticket transaction scenario mentioned above, Mr. Alpha's description of the goal was: *I issued five tickets to a passenger*, while my description of the goal was: *Mr. Alpha completed a mental mathematics transaction*. Elsewhere it has been documented in detail, specific mental computational activities of bus conductors as they dealt with multiple ticket transaction scenarios (XXX & XXX, 2008). I will now exemplify the use of Saxe's model to explore in depth perceptions of the mathematical goal associated with this ticket transaction activity (refer Figure 1).

I propose that Mr. Alpha performed a transaction by engaging in the following *mathematical activity* mentally: find the value of 50 – (3.50 * 5). This was done in two steps. First Mr. Alpha calculated the value of 3.50 * 5. He then subtracted this number from 50 to get the solution.

Mr. Alpha's *interactions* with the passenger were very crucial to this whole process. It was the passenger who posed the problem to the conductor in the first place. In this situation, the passenger helped Mr. Alpha in reformulating the problem by giving him extra money.

Mr. Alpha's *prior understandings* about whole number and decimal operations, and his prior experience working with money greatly helped him solve this problem. According to his reporting, this is how he calculated the value of 3.5 * 5: He first doubled 3.50 to get to 7.00. He further doubled 7 to get to 14. He then added 3.5 to 14 to get 17.50 as the solution. The next step was to calculate 50 – 17.50 At this point, he used his prior experience with money-related transactions to get the answer. He knew that 17.50 was 2.50 short of 20 *rupees.* Adding another 30 *rupees* gave him 50 *rupees*. Thus he added 2.50 to 30 to get 32.50 as the final answer. In this situation, he added another 50 to this and arrived at an answer of 33. The fact that he took another 50 *paise* from the passenger forced us to modify our original problem to the following problem 50.50 – (3.50 * 5) to which the answer is 33.00. Mr. Alpha reformulated the given problem into a new problem, used whole number facts, doubling, and compensating strategies effectively to solve the problem and complete the transaction.

Currency and printed tickets with denominations served as *artifacts* that helped Mr. Alpha complete this mathematical activity with ease. Mr. Alpha used the time to locate and tear out five tickets to mentally calculate the fare for five passengers. The passenger handed out a 50-*rupee* note, which Mr. Alpha knew was equivalent to five ten-*rupee* bills. As explained above Mr. Alpha reformulated the original problem and then thought of 50.50 in terms of currency – Five ten-*rupee* bills and one 50 *paise* coin. He then had no trouble locating and giving back 33 *rupees* back to the passenger.

Discussion

Every day, Mr. Alpha reported to work with several work-related goals in mind. These goals were fixed and were in line with bus conductors' responsibilities at work. In order to aptly fulfill these goals, he executed several work-related activities. Since these activities are geared towards fulfilling the work-related goals, I call these goal-directed activities. Based on my observations, field visits, and interviews with Mr. Alpha, I identified specific goals and goal-directed, work-related activities that required conductors to do mental computation (Table 1).

Table 1

Relationship between Overall Goals and Mental Computational Activities

Overall Goal	Goal-Oriented activities	Mental Computational Activities
Authenticate every passenger's travel in the bus by completing ticket transactions.	Approach passengers and gather information. Issue tickets quickly and efficiently.	Solve mental computational problems associated with ticket transactions.

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Mr. Alpha completed numerous ticket transactions during his bus trips. He associated an emergent goal with each of these ticket transactions. His description of these emergent goals was purely work-related and my descriptions of such emergent goals were strictly mathematical. Thus we held concomitant perceptions of emergent goals associated with ticket transactions (Table 2).

Table 2

Concomitant Perceptions of Emergent Goals

Activity and Source	Conductors' Perceptions	Researcher's Perception
Ticket transaction. (Refer to Snippet)	I issued five tickets to a passenger who travelled from point P to point Q.	Compare PPF (Rs. 50) with ATF (3.50 x 5)

Below, I explore both conductors' and researcher's perceptions of emergent goals associated with mathematical activities using Saxe's four context-related parameters: activity structures, conventions and artifacts, social interactions, and prior understandings.

Activity/Activity Cycles

In order to complete successfully the emergent goals associated with ticket transactions (mental mathematical activities), Mr. Alpha completed a series of tasks. The structure of an activity refers to a task or a series of tasks that he carried out to complete that activity. To authenticate every passenger's travel in the bus, Mr. Alpha executed ticket transactions. As an example, I present the activity cycle related to ticket transactions in Figure 5. This cycle could be conceived of as a three-step cyclic process.

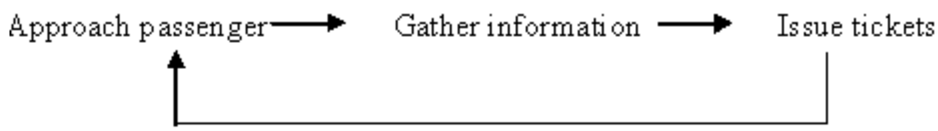

Figure 5. Activity cycle for a ticket transaction

For Mr. Alpha, the principal motive in this situation is to issue proper tickets to each passenger travelling in his/her bus. The researcher-identified mathematical goals are inextricably connected to this motive. The mental mathematical tasks that Mr. Alpha executed could be characterised in terms of a simplistic four-phase cyclic process. In the Figure 6, ATF denotes the actual ticket fare, PPF denotes the passenger paid fare, and BA denotes the balance amount due to the passengers.

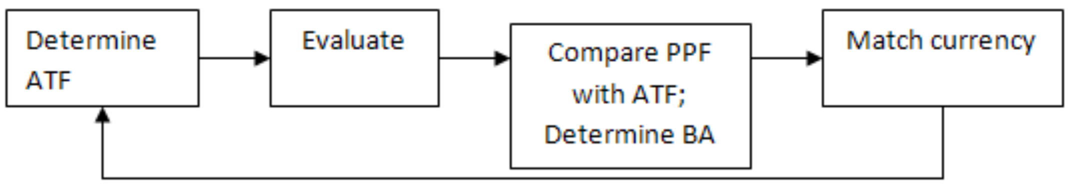

Figure 6. Ticket Transaction: Four-phase cyclic process

Social Interactions

Mr. Alpha's social relationships with drivers and the passengers played an important role in formation and execution of emergent goals. One interesting feature that was noted in this study concerned the role of participants in these social exchanges. Mr. Alpha continuously interacted with his passengers to gather travel information, suggest alternate exit points if necessary, indicate points of exit, and collect and issue the balance amount (difference between the PPF and the ATF). Although the conductor was the main actor in these scenarios, passengers' roles were equally important. Passengers provided the required travel information to the conductor.

In other studies, workers have mainly interacted with co-workers or their immediate superiors, but in this study, most of the social exchanges took place between conductors and passengers. Magajna and Monaghan (2003) observed an atmosphere of mutual respect between the technicians and other participants in a CAD/CAM workshop. In the present case, I found evidence to the contrary. Although Mr. Alpha addressed and treated all passengers with respect, nevertheless, he felt hassled when some passengers, occasionally, refused to cooperate with him. If the bus arrived behind scheduled time, or if the conductor stopped the bus before fare stages to close stage, or when the conductor requested small currency denominations in place of large ones, passengers sometimes became upset with the conductor. Overall, Mr. Alpha's interactions with passengers were friendly and helped him to complete his emergent goals efficiently.

Conventions and Artifacts

On every work-shift, Mr. Alpha engaged in routine activities that included finding out their bus route and bus service, collecting artifacts, identifying his bus driver, and identifying the timekeepers at the origin and destination. It was also customary for the conductor to approach passengers to initiate ticket transactions. Conductors used tools specific to their workplace – currency forms, printed tickets, and official records to create and complete emergent mathematical goals. The use of printed tickets helped conductors keep track of ticket fare calculations. The availability and the use of monetary units helped conductors create and complete mathematical tasks, and the goals associated with these problems, correctly and quickly.

Prior Understandings

Mr. Alpha's prior understandings included knowledge about (a) the bus routes, (b) the names and number of bus stops and fare stages along their bus routes, (c) stage fares and ticket denominations for different types of buses, and (d) school-learnt mathematical strategies. Mr. Alpha used some or all of the above-mentioned features to generate goals linked to the tasks set in activity cycles. Depending upon the task under consideration, he drew upon one or all of the resources to complete the task. Saxe (1991) noted that the knowledge that "individuals bring to bear on cultural practices both constrain and enable the goals they construct in practices" (Saxe, p. 18). Researchers have documented that carpet layers' and high school teachers' prior understandings of certain mathematical ideas constrained their ability to generate practice-related goals (Monaghan, 2004; Masingila, Davidenko, & Prus-Wisniowska, 1996). In some other studies, researchers did not find such evidence (Magajna and Monaghan, 2003). Although I documented several instances where conductors' prior understandings helped them construct emergent goals, I could not point to instances where their prior understandings constrained construction of emergent goals.

Conclusion

For this research project, I drew upon research on mental computation and everyday mathematics and extended it to a new context – bus conductors' workplace in India. In order to fulfill the demands of the workplace, Mr. Alpha had to complete all of their work-related activities. To complete some of the overall work-related activities he engaged in mathematical activities. In the absence of technological devices at his workplace, Mr. Alpha resorted to mental computation to solve computational problems. He was duty-bound to complete the goal-directed activities to his superiors' and his passengers' satisfaction. To do this well, he had to mentally calculate quickly, correctly, and efficiently. Thus the goal-directed activities not only demanded that Mr. Alpha engage in mental computational activities but also to complete the mental computational activities to everyone's satisfaction.

At the same time, it should be noted that the Mr. Alpha's workplace activities did not require him to engage in creative problem-solving activities. Since typical problems associated with the completion of goal-directed activities were limited to calculation of ticket prices and daily allowance calculations, Mr. Alpha solved only these types of problems at work. Hence I conclude that the goal-directed activities limited the range of computational problems and overall problem solving situations that were required of Mr. Alpha and hence the bus conductors.

Mr. Alpha's workplace mathematics has certain unique characteristics that are shaped by the context and the tools specific to his workplace. These characteristics were described in detail in the previous section. When Mr. Alpha completed ticket transactions, he drew upon their understanding of work-related notions such as determination of fare stages, fare stage numbers, and ticket prices associated with fare stages. This understanding, which I term *work-specific knowledge*, helped him determine the ticket fares and the balance amount due to the passengers and thus execute ticket transactions smoothly.

As a researcher I observed and acknowledged that at work Mr. Alpha established and achieved goals that involved significant mental mathematical activity. However, according to him, these goals were just the demands of his work place and part of his job. On hearing the primary researcher's interpretation of his work-related goals, this is how he responded:

You may be right. However, when I work, I am completely focused. I don't think about what mathematics I am using. I see money, I hear words, my hands deal with tickets and money and I calculate in my heart. There is little time to think about anything else. I cannot afford to make mistakes in my calculations because I may end up losing [the] department's money. So I only concentrate on doing my job quickly and correctly.

I wish to zoom back on ethnomathematical research studies that investigated the mathematics embedded in cultural practices. People who are engaged in these cultural practices did not consider “themselves to be engaged in doing significant mathematics” (Presmeg, 1998, p. 328). It was the researchers who interpreted their cultural practices and brought out the mathematics embedded in their practices. The same idea is underlined in the following definition of ethnomathematics proposed by Barton (1996) “Ethnomathematics is a research programme of the way in which cultural groups understand, articulate and use the concepts and practices which we describe as mathematical, whether or not the cultural group has a concept of mathematics” (p. 214).

Differing perceptions of Mr. Alpha’s work-related goals are complementary, and completely consonant with Saxe’s four-parameter model. The bus conductor’s mental mathematics is invisible to him in the press of his practice; in this researcher’s perception it is an example of ethnomathematics.

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