A Study of Estimation by Professionals at Work

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Abstract

Estimation is a learned skill that supports learning and applications in measurement as well as other quantitative contexts. Estimation is an important skill to learn in that it provides a foundation for measurement – it helps one determine the most appropriate tools for measuring specific items. Estimation is equally important as it helps one determine whether or not an actual measurement obtained is reasonable for a given context. It can be beneficial for learners to explore the application of estimation so that they have context and motivation for learning to estimate. The authors engaged with selected professionals during the course of their work to obtain information about when they use estimation, how they use estimation, how they determine if estimation is helpful and when they decide not to use estimation. The authors share estimation activities of the professionals, which result in "estimation declarations" and suggestions for the teaching of estimation.

Introduction

Estimation provides the foundation for measurement because estimation skills help us to determine the specific units or tools people will need to obtain a measurement. For example, an estimate of a length will help one to determine which tool (e.g., ruler, meter stick, measuring tape, trundle wheel, etc.) is most appropriate for determining the actual measurement. Estimation also helps us make decisions about whether or not a measurement obtained is reasonable for a given context (National Council of Teachers of Mathematics, 2000). Hence, estimation is both a precursor to and a foundation for measurement. In addition, "when people do not have specific information or need a precise answer, they often make an educated guess. An estimate is an educated or reasonable guess based on information, prior knowledge, and judgment. Even when information is known, the situation may call for an estimation" (Kennedy, Tipps, & Johnson, 2008, p. 175).

Estimation

Reys, Lindquist, Lambdin, and Smith (2009) define estimation as "…a process of producing answers that are close enough to allow for good decisions without performing elaborate or exact computations" (p. 225). In addition, Kennedy, Tipps and Johnson (2008) indicate that while "estimation is a reasonable guess, hypothesis, or conjecture based on numerical information", the estimate itself also "…communicates numerical information" (p. 173). While it is important to provide strategies to empower students to estimate, it can be very challenging to incorporate meaningful and real-life experiences from which these strategies emerge. To many learners, estimation is abstract and unrelated to "real" activity. "To produce an estimate is…a very difficult task for young children. They do not easily grasp the concept of "estimate" or "about" (Van de Walle, Karp, & Bay-Williams, 2010, p. 141). To this end, it is beneficial for learners to engage in discussions around issues that use such terms and or phrases as "about", "more or less than______?, "closer to ________or to________?" (Van de Walle, Karp, & Bay-Williams, 2010, p. 141). Another perspective of estimation is the determination of a "…range within which the solution [or estimate] might fall" (Hatfield, Edwards, Bitter & Morrow, 2008, p. 107).

Many students struggle with building the skill of estimation because it is more complex than responding with a suggestion isolated from conceptual understanding of the mathematics. In fact, "students who are proficient at written computations are not necessarily good estimators" (Reys, Lindquist, Lambdin, & Smith, 2009, p. 225). According to Bassarear (2008)

> The availability of inexpensive calculators has profoundly changed the way we compute. Most people now rely on calculators when exact answers

> are needed. However, estimating skills are still very important in cases where an exact answer is not needed and to the check the reasonableness of results obtained on the calculator. Estimation, in turn, requires good mental arithmetic skills, which come from an understanding of the nature of the operations, a firm understanding of place value, and the ability to use various properties (for example, the associative and distributive properties) (p. 143).

Recognizing this deficiency students often have with estimation, the authors determined to uncover how the concept and skill of estimation are addressed by professionals who estimate in the context of their work. The intent of the work is to disseminate these contexts so that teachers and teacher educators can have thoughtful examples of the application of estimation to provide for students and to use as contexts for instruction regarding estimation. The framework for the exploratory study rests with Ubiratan D'Ambrosio's notion of Ethnomathematics. According to D'Ambrosio (2007), "the great motivator for the research program known as ethnomathematics is to seek to understand mathematical knowing/doing throughout the history of humanity, in the contexts of different interest groups, communities, peoples and nations" (p. 8). This was the impetus for the authors exploring professionals engaged in their work that proposed to involve the act of estimation.

Methodology

Selecting Professions and Participants

In considering the work of estimating, the authors discussed professions thought to use estimation as a function of their work and collaborated on determining the professions to include. The authors concluded deliberations with 16 possible professions. Once the authors identified the professions that would receive focus, the authors scanned the community for representative persons for those professions and proceeded to call each professional to request each professional to engage in an on-the-job interview to observe the professionals at work. Of primary concern was to be able to observe professionals at work. The authors held the position that "the natural setting is the place where the researcher is most likely to discover, or uncover, what is to be known about the phenomenon of interest" (Maykut & Morehouse, 1994, p. 45).

All of the professionals were adults over the age of 21. There were 12 males and 5 females. The authors interviewed and observed 17 professionals engaged in the work of their profession (two shared the task of one profession.). Several of the professionals were known to one of the authors but not to the extent that the professionals needed to be removed from exploration. This current paper includes only four participants (White males, average age of 40) from the larger study.

Data Collection

The authors developed an interview guide that consisted of the following questions:

1. In what ways do you use mathematics on a daily basis to perform your work duties?
2. Specifically, how do you employ the skill of measurement?
3. In measuring, when do you use the skill estimation?
    a. What do you estimate?
    b. How do you estimate?
    c. When do you estimate?
4. What makes an estimate good or acceptable for particular professional tasks?
    a. Why don't you estimate? (That is, what situations lead to a decision not to estimate?)

The authors audiotaped the interview as well as collected the responses and field notes in writing as the participants spoke. Each audiotaped interview was scheduled for one hour at the place or site of employment for the profession so that the author could also observe the professionals at work with the expectation to observe the professional engage in estimation. The sessions lasted an average of 90 minutes. Several interactions lasted longer because of the nature of the profession and also several professionals preferred to talk as they worked rather than stop working to just engage in the interview. The author engaged the professionals in the interview as they worked, with the professionals often demonstrating something they were saying; this also provided prompts for questions of clarification about the professional activity. Visual observation data was recorded by still photography.

Data Analysis

One author transcribed each videotaped interview, using written notes for clarification when needed and possible. Both authors independently reviewed the participants' responses and chunked the data from each participant by responses to the interview question. This naturally led to the following categories of data.

1. Estimation tasks the participants engaged in for a task
2. Estimation strategies the participants used for a task
3. Participants' reasons for estimating and determining a good estimate for a task
4. Participants' reasons for not estimating for a task

The authors then collaborated to identify the salient points from the participants' voices regarding their engagement in estimating for their work tasks. Furthermore, the authors engaged in reading the participants' responses to search for patterns related to the participants' actions of measurement and estimation during the work experience. The authors coded data that related to identified patterns and then searched for sub-themes. Using these sub-themes, the authors revisited the data and condensed the sub-themes to result in a set of "estimation declaration" that would have implications for teaching estimation.

Results

The Traffic Patrolman

The traffic patrolman attended a two-day radar school to learn how to estimate the speeds of traveling vehicles. His instructor drove vehicles past the patrolman and fellow trainees at a set speed. They would then estimate the speed of each vehicle. After the simulation, the instructor would reveal the actual speed of each vehicle. The patrolman admitted: "You mess up, but over the two days, you practice all day long. You eventually get to the point to where you can make a guess." During work hours, the patrolman will often park his patrol car, watch a couple of vehicles pass by and make estimates about the speed of the vehicles. If he gets several vehicles that "look like" they are going 35 mph and the radar supports that, then he knows what 35 mph looks like in that particular setting.

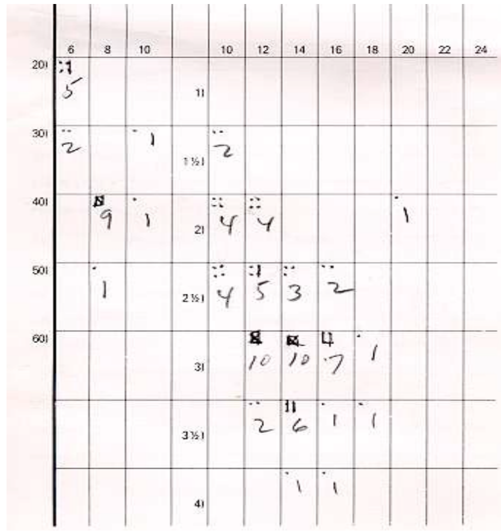
According to the patrolman, white vehicles, larger vehicles, and vehicles traveling through a curve will look like they are going faster than they actually are. During the interview the patrolman estimated a school bus' speed at 40-43 mph, but the radar speed registered at 38-39 mph. For him, his estimate was acceptable because it was within 5 mph. He is confident that he has developed his estimation of speed of vehicles through practice and experience to the extent that on a "good day", he can usually get within 3-4 mph of a vehicle's actual speed.

The Timber Sales Consultant

The timber sales consultant estimates several factors such as (1) size of tracts of trees in acres, (2) quarter-acre circular plots within tracts, (3) diameters of individual trees, (4) heights of individual trees, and (5) the number of board feet within a tract. Here are two of his estimation experiences:

To estimate the height of individual trees, he has to consider that "most sawmills want you to cut off the height estimate at a six-inch diameter top." Since the tree tapers in size as it grows, he stops to get his height estimate at the point where he thinks the diameter of the tree is down to six inches. For a tree that fell in the 14-inch diameter class, he calls "this tree a fourteen three. Three meaning there are three 16-foot cuts that I can get up to a six-inch top. So I'm assuming that up there at 48 feet, there is a six-inch top, so that gives me three 16-foot cuts." In terms of height, he classifies the trees in terms of half logs and logs, where every 16 feet is one log. So, fourteen three also provides the information that the tree is three logs in height.

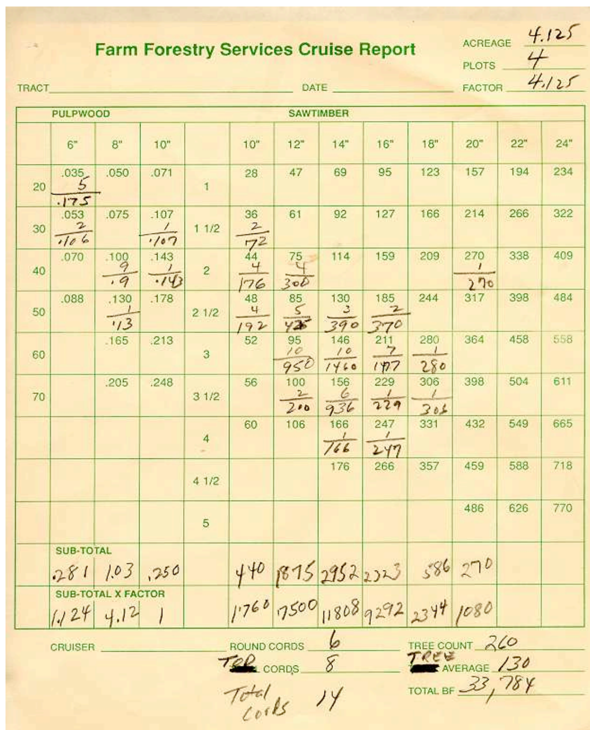
"I had a little tool, a hypsometer, that you could back off a chain (66 feet), shoot the bottom of the tree, then turn it and shoot to where you think the tree has a six-inch top. It's pretty accurate in giving you a height measurement. That was back in 1972 that I used it." Through repetition, he no longer uses a hypsometer. He is accurate enough to visually estimate the height of the trees. When he estimates the heights of trees, he states, "I'm not even thinking of it in terms of feet. I'm thinking about it in logs. I'm looking at it thinking three, three and a half, two and a half." In other words, he looks at the tree and immediately classifies the tree as a three-log (48 foot) tree, 3.5-log (56 foot) tree, etc., and he uses how he classified other trees as a way to check his current estimate. As he "cruises" a tract of land, he will categorize trees on a tally sheet (Photograph 1).



Photograph 1

Tally sheet used for placing trees in categories during the "cruise" of acreage

The final cruise report (Photograph 2) has the same tree categories as the tally sheet used during each cruise. At the end of the year, all of the cruises performed by him are tallied. His aim is that his total estimate of volume (board feet) for the timber cruised comes within 3% of the actual total board feet cut out from the trees. He emphasizes getting a good acreage estimate and a good diameter classification. The good acreage estimate is most important because "missing the acreage will blow the entire cruise." He focuses on getting a good diameter classification because an error in diameter classification throws off the board feet volume much more than an error in height classification. For example, a three-log, 16-inch diameter tree will have 211 board feet, but if it is incorrectly placed in the 14-inch diameter class, then the estimate will be 146 board feet, or 65 board feet too little. On the other hand, if the same three-log, 16-inch diameter tree is incorrectly placed in the 2.5 log category, then the estimate will be 185 board feet, or only 26 board feet too little. As he drives up to the tract, he gets a general sense for how many board feet per acre the tract of trees will have. Then, "when I'm going through there, I get a feel for how much there is per acre." As he is cruising the tract of land, he starts thinking, for example, "I'm on 7000 board feet per acre." According to him, "if it's normal South Georgia timber, there's going to be 6000-8000 board feet to the acre." He uses this up-front "feel for it" to verify his estimate in the end, to sense that he has a good estimate.



Farm Forestry Services Cruise Report

ACREAGE 4.125
PLOTS 4
FACTOR 4.125
TRACT ____ DATE ____

	PULPWOOD				SAWTIMBER							
	6"	8"	10"		10"	12"	14"	16"	18"	20"	22"	24"
20	.035 5 .175	.050	.071	1	28	47	69	95	123	157	194	234
30	.053 2 .106	.075	.107 1 .107	1 1/2	36 2 72	61	92	127	166	214	266	322
40	.070	.100 9 .9	.143 1 .143	2	44 4 176	75 4 300	114	159	209	270 1 270	338	409
50	.088	.130 1 .13	.178	2 1/2	48 4 192	85 5 425	130 3 390	185 2 370	244	317	398	484
60		.165	.213	3	52	95 10 950	146 10 1460	211 7 1477	280 1 280	364	458	558
70		.205	.248	3 1/2	56	100 2 200	156 6 936	229 1 229	306 1 306	398	504	611
				4	60	106	166 1 166	247 1 247	331	432	549	665
				4 1/2			176	266	357	459	588	718
				5						486	626	770
SUB-TOTAL	.281	1.03	.250		440	1875	2952	2323	586	270		
SUB-TOTAL X FACTOR	1.124	4.12	1		1760	7500	11808	9292	2344	1080		

CRUISER ____ ROUND CORDS 6 TREE COUNT 260
TOP [redacted] CORDS 8 TREE [redacted] AVERAGE 130
Total Cords 14 TOTAL BF 33,784

Photograph 2

Example of final cruise report

The Farmer

According to the farmer, "when you have to work with Mother Nature, the conditions really do make a difference in how you estimate." Yet, he still has to do the best he can for his job. He estimates factors like (1) the amount of seed to use, (2) sizes of planting fields, (3) when to plant, and (4) when to harvest. In order to determine the amount of seed to use, he uses lessons

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he learned when he farmed with his father. “My daddy knew how many acres were in the field and we would plant about half of it. We had to plant at least a quarter to half of the field to know. That would still give a guess, because you had to guess where half of the field was.” From this planting, they could then determine how much seed would be needed to finish planting the field. He now has an acre counter (wheel) mounted on his planter. It is a wheel that rolls out digits like an odometer. He multiplies the distance on the acre counter by the row width. This gives him the number of acres planted, and he uses this tool as part of his seed estimate. He fills the planter “level full” with seeds and starts planting the field by making one round through the field. “If that round is one acre or a half-acre, then you estimate how much seed you have used.” To determine the amount of seed used, he fills the planter level full again and estimates how much is left in the fifty-pound bag. This gives how much seed is being used per acre after making just one round in the field. Since he knows the size of the field, he can then estimate how much seed is needed to finish planting the field.

“Most all fields have been measured by the [state farming] office. You can get a number there, but it's not always accurate. For example, fence lines grow into the field. The trees and bushes growing out into the field will make the field a lot smaller even in a few years. You put fertilizer out there and the trees really grow fast. So if you have the acreage, you still have to estimate. You take two or three acres off a 50-acre field. After planting one time, I go with what the acre counter says. I do all of my own work, so I'm out in the open all of the time looking at land. I know what size my fields are and I'm looking at them all of the time.”

He also estimates when to plant based on the ground temperature. He plants at about 65^{o}F, and he obtains the ground temperature from reports. He must also look at the weather that is developing. An upcoming cold front will delay planting. “You just know when to plant. My daddy used to say when you kicked the covers back it was time for planting."

He knows how long it takes to harvest a set number of acres, but conditions vary, so he still has to estimate. If peanuts have been rained on, for example, then it takes a lot longer to harvest. His equipment now gathers six rows at a time, but other factors such as the condition of the equipment impact the rate of harvesting individual fields. "…You have to estimate down time, because you can count on that. For new equipment, you can estimate less down time." All of these factors impact how long the harvesting process will take.

"I used to think that you needed to get within 10 or 15 percent of my goal when estimating. The way things have been going lately on the farm, if I get within 25% of my goal then I'm doing really good."

The Automotive Technician

The automotive technician started off using measuring devices, but then through experience and instruction, learned to estimate. He credits his estimation abilities to the repetition of measuring with a device early in his career. He also credits instruction from others. Rather than go grab a measuring device, he was instructed to "look at this, or feel this." Early on in his job, he "relied on things like a gauge, the actual measuring device, until you get to the point where you can look at it and say 'this is where you are at.' Now I can go actually measure it, but I'm telling you this is where you are at." He does not need to measure any more. It took him "at least two years" to get to this point, but "that doesn't encompass the entire spectrum of what you are doing. I would say at least a third of what you are doing in that two- year period you would be comfortable enough to make an estimate without going to get a measuring device." He uses other common objects at times. He demonstrated that if you put a penny upside down in the tire tread and "can see all of Lincoln's head (Photograph 3), then you have less than 3/32 of an inch and your tire is worn out."

Photograph 3

From the top of Lincoln's head to the top of the coin is 3/32nd of an inch.

Estimation Declarations: Implications for Teaching Estimation

When the authors analyzed the interview data in search of specific estimation enactments, the authors found many and determined to classify them as "estimation declarations". These declarations emerged from the shared work of estimation by the traffic patrolman, timber sales consultant, farmer, and automotive technician and can be used to support the teaching of estimation.

1. There are distinct and real-life contexts for exercising estimation skills and these contexts are fruitful for exploration beyond abstract examples of estimation. Consider the four professionals.
2. People who are skilled at estimation have spent a great deal of time in practice and repetition of their skills, and they use these prior experiences to develop estimations.
3. Watching and receiving instruction from others with more estimation experience are important for building estimation skills.
4. Estimation within particular contexts relies on factors of that context (e.g., variations, average or usual measurements, characteristics of objects being measured).

5. Different estimation skills exercise different senses and body activities (e.g., intuition, sight, walking paces).

6. Practice and comparison with actual measurements first helps build estimation skills until one "gets a feel" for the task at hand.

7. Acceptable or good estimates vary with the context in which the estimates are developed.

8. Some estimates are more critical for a given situation than others. That is, some estimates, if too high or too low, may have a greater impact on a situation than other estimates that are too high or too low (e.g., planting an appropriate amount of seed).

9. In many instances, factors that cannot be controlled (e.g., nature or natural phenomena) impact the outcome of estimates.

10. Within given contexts, estimates may change as other factors change (e.g., size of farming fields).

11. Personal and cultural traditions carry valuable information that can be useful for estimation (e.g., the farmer's father's advice about planting).

12. Knowing informal tools that are useful for estimation (e.g., coins) can benefit estimation activities.

Knowing and doing mathematics is manifested in many ways, including in the work of people as they perform required tasks. The estimation declarations presented represent just some of the insight gained from the interviews and from accompanying the professionals on their jobs. Some implications for pedagogical practice in teaching estimation include the following:

- Share information about how professional use measurement estimation to perform their job duties as a way of connecting classroom instruction to real-life experiences. Just being about to share these interviews with students have helped to capture their interests in estimating in the context of measurement.

- Invite professionals to speak about and demonstrate their estimation experiences and skills. This will help students see the usefulness of estimation in real-life contexts.
- Seek opportunities for students to "shadow" professionals as they perform job duties that involve estimation.
- Provide ample opportunities for students to repeatedly practice estimation.
- Students can conduct interviews of professionals in their local communities to gather information about how these professionals use estimation in the course of their work. Students can follow up with written reports and demonstrations.
- Encourage students to develop informal estimation benchmarks. For example, the automotive technician uses a penny to support his work.
- Design simulations that model the ways professionals estimate on their jobs. For example, the estimation practices of the traffic patrolman can be modeled inside or outside using moving objects (e.g., students pushing toy cars or navigating remote cars, students riding bicycles, etc.) and stopwatches.

Conclusion

The skill and enactment of estimation comes to life across many professions as when examining the act of doing mathematics when professionals engage in their work. Missing from students' mathematical experiences is often a sense of what doing mathematics means outside of the classroom context. Approaching mathematics via the lenses of ethnomathematics reveals ways in which teachers can present more meaningful mathematics contexts to students and empower them to engage in the ways of doing mathematics.

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