Relationships Between the Hopi Calendar and Measurement Concepts

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Abstract

There is much to be relearned about mathematics by seeing it from a viewpoint of a particular society. This paper describes measurement concepts by illustrating how the ancient Hopi used them to actively mathematize their environment, building a calendar for celebrations and festivals necessary for their societal mission. The core mathematical concepts are: size, standard unit, zero-origin, tiling, continuousness, and part-whole. The Hopi calendar is presented in terms of the importance and development of the moon, as a counting unit. Ultimately, a 13th moon helps explain the Hopi calendar and further clarifies the mathematical concepts.

Introduction

Mathematics develops in response to humanity's efforts to better understand the environment. However, that humanistic intent is not always reflected in the mathematics of the classroom. Fostnot and Dolk (2002) posit that for years, mathematics has been taught as if it were a dead language. That is, something to be remembered for its own sake with little recognition of mathematics as a continually changing entity, known uniquely by each person and reflective of the times. Fortunately, researchers (e.g., Zaslavsky, 1973; Gerdes, 1998; Eglash, 1999) in the discipline of ethnomathematics are continually revealing mathematics through new eyes by reconnecting culture(s) to mathematics.

While preparing lessons about basic measurement concepts, we encountered such a journey. There is much to be relearned about measurement by seeing it within a viewpoint of a particular society. Understanding the construction of and the modification to the Hopi Festival Calendar is rich territory for illustrations of measurement content. I was not looking for something unique to "introduce" measurement. As Powell and Frankenstein (1997) warn,

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culture(s) must genuinely permeate mathematics lessons. Classroom discussions must not simply cite a few cultural connections, as an aside or a precursor that takes place before the "real" mathematics is presented. With this in mind, Barton's (2008) advice encouraged a deep study of the Hopi *system* of knowledge to try to draw parallels between their cultural practices and the mathematics (in this case, measurement) at hand.

Despite the initial expressed purpose of improving measurement instruction, I immensely broadened my personal mathematics knowledge. This new way of understanding measurement had a profound impact on the way I now understand and teach about measurement. It seemed important to share the information that was uncovered. What follows is a record of observations along the path I charted and the connections I have seen. This paper describes measurement concepts illustrating how the ancient Hopi used them to actively mathematize their environment and to build a calendar for celebrations and festivals necessary for their societal mission.

Ethnomathematics

In 1990, D'Ambrosio catapulted the field of mathematics forward by suggesting mathematics is a system of knowledge that should be understood within the context of social and cultural knowledge. In so doing, he questioned the prevailing paradigm that mathematics is static, antiseptic knowledge, devoid of cultural connections or influences. Knowing mathematics *includes* having awareness that it is a product of ongoing (and thus changing) efforts to explain and cope with an environment. Mathematics is created by and known by members of the society dealing with their environment, so is influenced by other knowledge from that culture as well as generations of knowledge produced by previous members. Understanding mathematics in this way is the field of study, ethnomathematics. This "programme … looks into the generation, transmission, institutionalization and diffusion of knowledge with emphasis on the socio-cultural environment" (D'Ambrosio, 1990, p. 369). More recently, D'Ambrosio (2006) suggested that ethnomathematics is more than recognizing mathematical ideas and practices of various cultural groups, it is deeper – it is also a comparison between several cultural groups.

Several ethnomathematics researchers (e.g., Powell & Frankenstein, 1997; Gerdes, 1998; Eglash, 1999; Barta & Shockey, 2006) continue to shed light on mathematics through careful study of cultural ramifications of mathematical ways of thinking. In some cases, researchers study familial cultural heritages, in other cases, they study other cultures. Some researchers physically immerse themselves, living in a culture different from their own, while others cull information from secondary and/or primary sources to reunite the mathematics with its cultural heritage. In all cases, the field of mathematics is enriched, as situations unique to various cultures are revealed to the global community through mathematical eyes in ways that enable researchers to study a multitude of views.

Eglash (1999) interviewed numerous African artisans and scientists as part of his efforts to deepen knowledge about fractals. He found systematic use of fractals in areas ranging from architecture to hairstyling across the diverse cultures of Africa. Zaslavsky (1973) documented the numeration concepts and procedures historically used among different African cultures. She describes a long process beginning with sifting through the literature of diverse disciplines ranging anywhere from African economics and anthroplogy to archaeology and linguistics. Then she followed the trail of her literary findings to personally interview African scholars, from many societies. Ortiz-Franco (1999) described the innovative ways various pre-Columbian cultures in the Americas created and used a positional numeration system not only to quantify their environment but to effectively mark time. Ancient Mayan astronomers predicted "solar and lunar eclipses to the day, hundreds of years in advance" (p. 221) and made these predictions hundreds of years before Galileo's birth. Ortiz-Franco pointedly challenges beliefs that ignore Chicano ancestors' contributions to mathematical knowledge such as beliefs that typically give credit of discovery of mathematics to civilizations, like those of Euclid and Aristotle, which actually emerged long after Mayan civilization. Barta and Shockey (2006) describe an historical account of the dynamic and personal ways the Northern Ute, a Native American people, use and know mathematics. Their work respectfully summarizes traditional Ute uses of a wide range of mathematical ideas and includes essential comments from tribal elders. Gerdes (1998) solicited example after example of unique ways of knowing geometry theorems among his secondary education students. They recollected numerous perspectives of the theorem, *diagonals of a rectangle are congruent* each related to specific cultures of their communities. After Moore (1988) studied Native North American petroglyphs, finding new ways to understand iteration, recursion and several other geometry ideas, he created meaningful classroom activities to illuminate these ideas holistically. As researchers endeavor to reconnect mathematics to the context of its socio-cultural environment, mathematics education has also benefited and grown.

Teaching and Learning

When viewing mathematics as indelibly linked to culture, teaching and learning mathematics takes on a different image than is often seen in school. Mathematics instruction becomes less of a chronicle of information (a static and unchanging set of rules and laws) and more of a dynamic problem solving investigation (something one *does* to handle an unexpected problem in an environment.) Changing instruction toward this view enriches learning, both because the mathematics is more tightly connected and because learners come to know mathematics while simultaneously honoring ways of knowing within the culture from which the problem originates. It is within this context that I study culture and mathematics. As part of preparing for instruction, I find it not only satisfying, but enlightening to ground the lesson explicitly by making a meaningful cultural foundation integral with the mathematics topic(s). When learners layer mathematical ideas with knowledge about world cultures, their overall web of knowledge, including mathematics, is more dense. Richly connected knowledge is easier to recall, easier to use, deeper, and more long-lasting (NCTM, 2000). So, it is a natural extension to mathematics education to study mathematical ideas (like measurement) through cultural knowledge of members of the society who practice it (D'Ambrosio, 1990; Gerdes, 1998).

As a place where mathematics and education meet (Miarka, 2012), ethnomathematics can provide meaningful opportunities for studying mathematical ideas, including the focus of this paper: measurement. This paper explains how and why the inclusion of a thirteenth moon solved a societal "problem" for many Native North American tribes, including the ancient Hopi. Drawing parallels to the Hopi moons reveals the complexity of deceptively simple measurement concepts and weaves them together in a exceptional way.

Measurement

Learning and Teaching

Learning and teaching about measurement are intricate activities because it is composed of several related concepts, all of which must be applied and coordinated to appropriately determine the size of something (Lehrer, Jacobson, Kemeny, & Strom, 1999). Basic measurement concepts discussed in this paper are size, standard unit, zero-origin, tiling, continuity, and part-whole.

Basic Concepts

Before learning to measure, children must gain a basic awareness that objects have a *size*. Knowing that some things are somehow larger, deeper, or longer than other things is fundamental for building a knowledge base. Young children can order objects from smallest to largest whereas older children can compare objects to one specific object. The specific object of a size comparison might be a pencil, a footstep, or even recess. By mathematizing their classroom environment, measuring familiar things against familiar units such as these, students make connections between and among the relative sizes of elements of their immediate environment. After ordering and comparing, asking, "*How much* larger, deeper or longer?" piques children's interest in the inevitable follow-up task of communicating their ideas about size to others.

To explain measurements to people outside of their immediate environments, children build awareness that communicating how much larger, deeper or longer an object is requires some sort of *standard unit***.** Instead of using a pencil, a footstep, or recess as their comparison unit, they develop conceptual understanding that standard units are things against which *everything* can be compared and about which *everyone* knows. Examples include inch, centimeter, second, year, degree, square yard, and cubic meter. Children are often interested in knowing that standards units like these exist beyond the classroom in every day life and that it is important for a culture or a society to agree on them.

Basic Procedures

Sometimes, the typical measuring device does not properly align with the item being measured. Flexibility in using measuring devices depends partly on the learners' conceptual fluency with the *zero-indicator* on the device. Several concepts (like standard unit) are automatically embedded in most measuring devices, but the zero-indicator's importance is in showing where to start the measurement process. For example, when measuring recess, children need to recognize that even though the clock says 11:20 upon return from recess and it said 11:05 upon departure, they did not spend 11 hours and 20 minutes playing. An adjustment of 11 hours and 5 minutes is needed, making recess 15 minutes long. The zero-indicator was 11:05. At that exact moment, in fact the exact second, academics ended and recess began.

Joining Concepts and Procedures

The idea that the standard unit of a minute is used over and over again and that a new minute begins the exact moment the previous minute ends is an example of the concept of *tiling.* However, there are also moments when it is appropriate to pause the tiling when measuring in certain situations. Knowing when to pause the tiling is an important piece of the tiling concept. For instance, reconsider measuring the length of recess bearing in mind that it takes children 2 minutes to walk from the classroom to the playground. Those 2 minutes are not counted as part of the recess nor as part of academics.

Whenever the tiling process is being applied, the process is repeated *continuously* until reaching the end of the object being measured. Counting the number of iterations is the same as describing the size of the object under investigation. Of course, it is possible that the size of the object being measured may not coincide with a whole number of repeated standard units. So, understanding the notion of *part-whole* is important to fully appreciate the nuances of measurement and the ways that measuring devices can be applied. For instance, in the measurement of time, we have half hours, tenths of seconds, or quarters of years. Coordinating all these concepts to execute a procedure requires multiple learning experiences in varying environments. Investigating the calendar of an ancient agricultural culture, with attention to measuring time, reveals these concepts in a copious and rich way and brings new life to understanding them within a significant context.

The Ancient Hopi

The Hopi are one of the Pueblo peoples who, for centuries dating well before the time of Columbus, have made their home on the mesas in Northeastern Arizona in North America. They lived in the community but traveled out from their homes to farm their carefully nurtured desert fields.[1] Successfully growing crops in an inhospitable desert climate severely depended on a balance of moisture. Respecting rain was an integral part of Hopi culture and a cornerstone of Hopi history. This aspect of ancient Hopi history and knowledge was one of the mores passed down through oral tradition.

Celebrations

Such storytelling kept history alive during the Soyal celebration. Those who gathered for the sacred stories listened intently as Hopi historians used corporeal words to weave stories into the spiritual atmosphere. In particular, they repeatedly told the timeless story about Hopi emergence into the Americas and credited their long-lasting existence as a result of having been chosen as the caretakers of Earth. This solemn responsibility (that so depended on rainfall) impacted nearly all Hopi culture. Cultural activities ranged from the material, planting corn seeds, to the spiritual, maintaining relationships with Katsinas, the spirit beings who guided them. The material activities relied on nourishment from Earth whereas the spiritual activities provided nourishment for the Earth. Naturally, it was of paramount importance for these activities to coincide with the rhythms and seasons of Earth. To further celebrate their role as

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[1] *Past tense is used throughout the remainder of the paper to base the discussion in ancient calendar-watching strategies. This is not meant to imply Hopi culture is not presently vibrant, nor that the ceremonies described in this paper are no longer practiced.*

nurturers of Earth, the Hopi constructed a framework of cultural festivals and celebrations to surround the hard work of tending for Earth. Certain agricultural activities corresponded with certain festivals and celebrations.

Forecasting celebrations. The responsibility to tend for Earth motivated accurate timing of these celebrations and festivals. Agricultural activities like planting and cultivating had to be carefully arranged, to coincide with the Earth's nourishments and needs. "…determination of the last freeze coincident with the vernal equinox is essential to crop survival" (Autrey and Autrey, 1981, p. 96). Planting beans or corn at the right time in the cycle, neither too soon nor too late was predicated upon appropriately forecasting the solar year (winter solstice to winter solstice). Embedding the Earth's rhythms in their cultural calendar reminded them of upcoming events. But, how did the Hopi elders schedule the community calendar? They matched celebrations and festivals with specific moons, each of which had a specific name. As those specific moons appeared, these ancient Hopi knew which ceremony and hence, which planting, tending, and harvesting activities had arrived. "Every ceremony is associated with a moon and every moon with a ceremony" (Ellis, 1975, p. 65). In other words, the moons measured time.

Measurement concepts of Moon time. The 29.5 day *standard unit* of the moon is itself a composite of phases. It moves from the first sliver of a moon, through waxing, on to a full moon, back through waning until just before the final sliver disappears. What exactly did the ancient Hopi need to measure with such a standard unit? To be sure, they needed to predict the Winter Solstice. But they could have done (and did) that by comparing their astronomical records with the position of the sun in the sky. When the sun's path is the shortest, it provides the least amount of daylight. The sun's shortest path also travels along its lowest path across the skyline, something visible even to the naked eye. The shortest and lowest path is the Winter Solstice, which marks the beginning of a new solar year. Certainly ancient Hopi could thusly predict this change over to a new year. And, this would have been enough if their cultural celebrations and festivals occurred only at the Winter Solstice. Because of the agricultural needs and responsibilities throughout the rest of the year, they needed to measure the solar year with some kind of smaller unit and they used the moon.

Appearance of that first sliver of moon, the crescent, marked the beginning of a new unit of moon time (Ellis, 1975). Determining placement of the *zero indicator*, or origin, is required in any measurement situation. In a Western[2] sense, this is akin to beginning the month of January at the stroke of midnight on December 31. Ancient Hopi began measuring moon time upon visual verification of that first sliver of new moon.

There is a two-day lull (sometimes called a *new* moon) between the last sliver of one moon and the new sliver of the next moon. Did ancient Hopi worry about this lack of a moon? "The two preceding days in which no moon is visible are not counted; they fall between lunar periods" (Ellis, 1975, p. 64). So, the Hopi *standard unit* for measuring the lunar year was actually approximately 27.5 days, even though the Western description of a lunar month is 29.5 days. For the demarcation of time using the moon unit, the Hopi simply omitted those two days. That is, cultural events tended not to happen during a new moon.

Conceptualizing time. The way westerners would worry about these two days illuminates conflicting subtleties within the general concept of time from the Hopi point of view when compared to the Western point of view. The Hopi did not "count" moons or days. Counting, as in moving toward infinity, implies a view of measuring time as one of *linear passage*. Instead, the

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[2] *The term "Western" is intended to imply a roughly European viewpoint and used throughout the manuscript to serve as a way to alert the reader if there is a need to translate a Hopi viewpoint into the cultural viewpoint of the Western author.*

Hopi view of time was one of *constant cycling*. Each moon unit was associated with a recurring event rather than as one moon in a linear series of "next" events. "Our "length of time" is expressed by the Hopi not as a linear measurement, but as a relationship between two events" (Waters, 1969, p.104.) In effect, they did not tally time as much as they experienced it. During the two-day new moon, there was nothing to measure. That is, there was nothing to experience during that intermission. Since the western perception of time is linear, these ideas required me to stretch my thinking. But for the ancient Hopi, time was naturally a cyclic and symmetric affair. "The Hopi think of the moons of their calendar as being in a continuous succession which we can best picture as a moving chain encircling a wheel, …" (Ellis, 1975, p. 69). They marked time not for calculating its *passage*, but more for properly aligning themselves with cyclic needs of the Earth. However, in so doing, they continuously tiled the *standard unit*, placing it end-to-end. But the calendar was not a line, it was a circle. They measured with the moon in order to connect two reappearing events, such as a moon and its agricultural celebration or festival. In so doing, they related the appropriate time during the solar year with the appropriate agricultural activity like planting or harvest.

Important events. There was a moon for Waiting (it is not yet time to plant corn, beans, and squash) and a moon for Planting[3]. There was even a moon, Sparrow-Hawk, designated as the time for taking account of celestial observations and guiding creation of the specific schedule for the upcoming year's community calendar (Secakuku, 1995). The (Autumn) Kelmuya season ceremony was always scheduled with Sparrow-Hawk Moon. In the Kelmuya ceremony, "singing is loud and deep and echoes the drumbeats, which are prayers for long and pleasant life for all

[3] *Moon names were compiled from a variety of references: Sekakuku, 1995; Wright & Roat, 1965; Ellis, 1975; James, 2000; http://www.wwu.edu/skywise/indianmoons.html#Hopi*

mankind, moisture in the form of snow and rain, and a bountiful harvest of nourishing crops" (Secakuku, 1995, p. 6). Kelmuya prepared the Hopi for the next moon, Sacred Moon, which brought the ever-important Winter Solstice ceremony, Soyal.

Activities of the Soyal ceremony included the storytelling, noted earlier, that kept history alive. It also set in motion the first scheduled event in the yearly cycle of caretaking. During this ceremony, the Hopi persuaded the sun to begin to lengthen and raise its path across the sky, slowly relieving Earth of those short, dark, but necessary, winter days (James, 2000), and creating the Winter Solstice. As if further authentication of the importance of this ceremony were needed, for the first time since the preceding Summer Solstice ceremony, a Katsina returned from the spirit world to walk among the Hopi people. Any effort to understand ancient Hopi culture would be incomplete without studying the role and importance of the Katsina. That said, a westerner's efforts to really comprehend the importance of the Katsina to Hopi culture will be incomplete due to the lifelong, dynamic nature of that relationship.

Katsinas. The depth of the relationship between Katsinas and the Hopi people must be understood as one where Katsinas are sacred "spirit beings to invoke substantial growth and maturity for all mankind" (Secakuku, 1995, p. 3). Since the Hopi were entrusted by the gods to provide watchful and careful nurturing of Earth, it was important they were given both spiritual support and earthly support (for instance, rain) to accomplish this directive. "Katsina dance events combine responsibility with pleasure because of the grave commission to provide moisture for the whole world" (Kealiinohomoku, 1989, p. 58).

For ceremonial purposes, selected Hopi people dressed like the spiritual Katsinas to symbolically represent the ethereal beings and to perform ritual singing, dancing and interactions with the people. Just as moons corresponded with ceremonies and agricultural activities corresponded with various ceremonies, Katsinas matched with specific ceremonies and dances. So, moons had not only to precisely match the Earth's rhythms, but also had to match the correct Katsina. Hosting the correct ceremony and thus interaction with the correct Katsina(s) signaled the correct agricultural activity for successful harvest and tending of the land. Correctly using moons to measure and plan these important events was imperative. Katsinas permeated ancient Hopi daily life in many ways. The Katsinas knew if the people had properly prepared for Soyal and upheld their promise to nurture Earth through another year (Secakuku, 1995). During a late winter moon, Purification Moon, Katsinas delivered a small bundle of bean sprouts to each family (Secakuku, 1995). These sprouts represented promises of the coming season. The importance of correctly timing ceremonies, so the appropriate Katsinas interacted with the Hopi people to nourish the correct agricultural activity, was the only way to ensure the Earth was properly tended.

With so much riding on the timing of the Soyal celebration, and with time being measured in moons, it was imperative and a somewhat delicate project to correctly match moons to the correct corresponding ceremony. Correctly timing Soyal, would mean the first sliver of Sacred Moon appeared *before* the days started to lengthen again, that is, before the turning point of the solar year, the shortest day, the Winter Solstice. Hopi Celestial Body-Watchers were expected to appropriately calculate this event beforehand. So, in effect, they had to be sure they knew when they were in the time of Sparrow-Hawk moon, when all subsequent times for ceremonies and festivals were planned (see Table 1).

Table 1: Hopi Celebration Calendar

Moons	***Celebrations/Events***
Sparrow-Hawk Moon (This moon is immediately prior to the moon during which the Winter Solstice occurs)	• Second initiation period for youths • Youths became warriors (hawks symbolize this achievement) • Make new fire • Leads up to Winter Solstice ceremony
Sacred (or Respected) Moon (Moon during which Winter Solstice *must* occur)	• *Lengthy* ceremony, Soyal, First Katsinas appear *must occur during Winter Solstice*
Play Moon Joyful Moon	• Gaming and entertainment (weather is too cold to work outside) • Social dances are held • Secretly-planted beans & corn are brought to heated area, for sprouts
Purification Moon	• First initiation period for youths • More Katsinas appear to insure fertility of soil • Pre-planting ceremony • Present bean & corn sprouts • Plant more seeds, nurture in heated area • Young children receive Katsina dolls
Whispering Wind Moon	• Plant "early" corn, so Katsinas will have it for the harvest

	celebration (much later)
Windbreaks for Small Plants Moon	• Seeds are prepared for planting
Waiting Moon	• Planting of non-corn begins (e.g., melons, beans & pumpkins)
Planting Moon	• Corn planting begins • Young people are taught how to farm
Homedance Moon Moisture Moon	• Rains begin • Home-Going Ceremony (Niman) Katsinas travel "home" *occurs after Summer Solstice*
Joyful Moon	• Snake/Antelope or Flute ceremonies • Butterfly Dance
Big Feast Moon Moon of Plenty	• Smoke meeting - women's societies bless the harvest season • A few crops picked for feast
Harvest Moon	• Harvesting begins • Harvest Basket Dance
Corn Husking Moon (If needed – some references do not include this moon)	• Burden basket is carried

The Problem

Ancient Hopi elders needed to know exactly which moon was shining so the community calendar would be accurate. A successful harvest required things, like planting, to happen at appropriate times in the solar (or agricultural) year. And, planting had to occur during Planting

Moon just like Winter Solstice had to occur during Sacred Moon. At first, it is tempting to think that keeping track of moons and reciting them in order would have been sufficient. However, when the ancient Celestial Body-Watchers compared the sun movement (solar year) to the moons, they found a small problem. The 365.25 day solar year (Winter Solstice to Winter Solstice) is not neatly divisible by the 29.5 day moon (new moon to new moon). There is a remainder of around 10 days: a solar year (365.25 days) is approximately 10 days longer than 12 iterations (354.9 days) of the moon. Simply counting out 12 moons would have resulted in incorrectly timed celebrations. Within a mere three years, an extra 30 days would occur, which encompassed an entire moon. We can see how quickly *tiling* the moon unit fell out of sync with the solar year and forced some sort of reconciliation of the *part-whole* concept. A solar year ends and begins partway through a thirteenth moon.

The Solution

So, approximately every 3 – 4 years, a 13th tiling of the moon could be fully completed before the height of the sun's path across the skyline alerted the Celestial Body-Watchers to conduct Sparrow-Hawk festivities. Sparrow-Hawk is considered the 1st moon of the cycle because this is the time when the year's festivities are announced. Sacred Moon, the moon bringing the Winter Solstice, is actually considered the 2nd moon in the cycle even though it's the first moon of the Solar Year. To correctly match the Winter Solstice ceremony to Sacred Moon means the ancient Hopi needed to figure out how to handle that part of a moon that caused a misalignment (every 3 – 4 years) between Sacred Moon and the Winter Solstice. This was compounded because they had no control over the *size* of their standard unit. Nor could they just pick up their measuring device and move the *zero-indicator* to the Winter Solstice and begin tiling again. They knew whatever decision they made to address the 10-day part of the 12th moon (or the 19.5 day part of the 13th moon) would have ramifications for the subsequent solar year.[4]

Connections to Measurement Concepts

The Hopi reapplied the *standard unit* over and over again, *tiling* it to measure time. They found a way to handle the *part-whole* problem: being unable to physically reset the *zero-indicator* to the Winter Solstice when the end of the 12th moon did not end in such a way that Sacred Moon would match up with the end of the solar year. This process fit well within the Hopi worldview of continuous symmetry within time. Just like they anticipated the reiteration of cultural events during each solar year, they also counted on Katsinas to return to them each Winter Solstice, after having been in the spirit world since the ceremony following the previous Summer Solstice. They saw things like the recurring existence of a *longest* day and a *shortest* day as a promise that continuity of time would be preserved. Rather than time passing, it recurred.

Here, I came to appreciate the deft manner in which ancient Hopi demonstrated deep understanding of measurement concepts in dealing with their problem. They inserted another standard unit when confronted with the part-whole situation that was required and could accommodate the situation (approximately every 3rd or 4th year.) Viewing calendar measurement the way the ancient Hopi perceived it brings new life to my understanding of measurement concepts. Merging Hopi thinking with my understanding of measurement concepts wrapped cultural respect and appreciation around mathematical knowledge while also growing deeper mathematical knowledge from it.

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[4] *It should be noted that several sources (Ellis, 1975; Wright & Roat, 1965; Secakuku, 1995); McCluskey, 1981) indicate that 20th and 21st century Hopi do not measure exclusively with moons. Now, they recognize a 5 month time (July – November) when Katsinas are in the spirit world and a 7 month time (December – June), when Katsinas walk among them, which represents adaptation of the lunar month standard unit to the 12 month constructed calendar imposed upon them by the western world.*

Several mathematics education researchers believe teaching mathematics from an explicit ethnomathematics viewpoint may address the fact that many K-12 teachers are from different cultures than their students (Fuller, 1992; Gerdes, 1998; Malloy & Malloy, 1998). It is not atypical for a teacher to teach a room filled with students of many cultures, possibly none of whom represent the cultural group in which the teacher himself or herself claims membership. The study of ethnomathematics is dedicated to understanding mathematical concepts through the eyes of members of a social structure in which the mathematics is being implemented, invented, or studied. People's ways of knowing are cast in molds upon birth. These molds are influenced, in fact designed, by the culture into which the person is born. Knowing mathematics is no exception. Clearly, two is always two, circles are circles, and time is something we can measure. However, *ways of knowing* these mathematical ideas are not so antiseptic. While ethnomathematics does not suggest that in some cases two is three or that circles somehow look like squares in some cultures, how we think about and how we know about "two-ness" varies according to the culture in which we are members.

To teach mathematics within a context of such diversity, we ourselves must relearn mathematical practices from a variety of cultural groups. By connecting mathematics learning and teaching to cultural knowledge (our own and others'), we possess a richer tapestry of mathematics knowledge, hence an opportunity for better understanding and teaching of it.

The Resulting Mathematics Lesson

The goals of my lesson were for students to: (1) identify the measurement concepts embodied by the moon, (2) use a circular representation of a lunar calendar (moon ruler) to recognize the value and pitfalls of measuring years with moons, and (3) describe how the Hopi adjusted for these issues as evidenced by a statement acknowledging that 12 moons or 13 moons each year would put calendars out of sync with the planting seasons and (4) describe that the Hopi handled this by inserting a 13th moon only every 3 – 4 years. My lesson plan opened with a discussion question designed to focus their comments on a need to pinpoint a specific date during a year. I asked, "What if you lived in ancient times, when there were no paper calendars. How would you tell someone when your birthday was?" During the resulting discussion, I watched for opportunities to key the conversation toward using Winter Solstice as a landmark time, the need to measure specific moments during a year, using the moon as a standard unit, and the issues that would arise by using moon phases as the standard unit to measure time. After the discussion, where students recognized the plausibility of measuring a year using moons, I issued students a circular moon-ruler and a circular year chart and asked them to measure the year with a 12-moon cycle. After the students used the rulers they saw how quickly a 12-moon count ran out of sync with the seasons. Twelve iterations with the moon acting as a standard unit account for only 349° of the 360° in the circular year.

I then described how the Ancient Hopi had to know when the moon for Winter Solstice would occur and that 12 moons occurring in less than one year complicated timing for that event. The Ancient Hopi felt responsible for tending the earth and it was imperative for them to know when the Sparrow-Hawk moon would occur. It absolutely had to occur at the appropriate time. It had to be the moon immediately before the Respected Moon – the moon with Winter Solstice. Winter Solstice accompanied the ceremony that asked Katsinas for critical help in tending the earth. I also described the Celestial Body-Watcher, who was responsible for knowing when to insert the 13^{th} moon, so that all of the year's festivals occurred at the proper time, with each of the corresponding moons. Students then read "The Problem" and "The Solution" before completing the Assessment. In the assessment, students calculated the first possible placement of

Corn-Husking Moon, described how the measurement concepts interacted with the moon ruler, explained why the Hopi needed to measure years and explained how the Hopi Celestial Body-Watcher handled issues with the moon ruler.

Summary

As a place where mathematics and education meet, ethnomathematics can provide meaningful opportunities for studying mathematics ideas. The field of ethnomathematics can provide appropriate contexts for studying the teaching and learning of mathematics. It recognizes social activity and cultural membership as important components of mathematics knowledge. Inspiring diverse groups of children to build knowledge on their existing understandings seems more deftly handled when related social and cultural knowledge is taken into account. Studying and appreciating the thicket of connections between mathematics and culture (like measurement and the Hopi celebration calendar) allows teachers like me to deepen my knowledge. In the end, this grounded my efforts to create lessons that helped students develop connected knowledge about teaching, learning, and measurement concepts.

Endnote

The Hopi have a long history and a complex culture. Differences in knowledge due to the personal nature of storytelling are inherent even among different Hopi villages and across the three mesas. "No published material will ever meet the general approval of the Hopi people" (Secakuku, 1995, p. viii) and the Hopi people tend to be a private society. Some westerners have been allowed glimpses of enduring Hopi ideologies and beliefs (e.g., McCluskey, 1981; Carlson, 1986; Pacific Arts Video Publishing, 1990; Page & Page, 1994; Fewkes, 2001). The author acknowledges her decidedly and inescapable "western" viewpoint and offers a consequential apology to the Hopi people, who know that non-Hopi people cannot fully understand Hopi ways of knowing. She has treaded as lightly as possible, gathering only enough information to create this lesson's background. She requests their patience with her efforts to appreciate and respect their ways of knowing, as she studies the mathematics of some of their ancestors for the purpose of enhancing mathematics education.

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