Thomas Storer’s Heart-Sequence:
A Formal Approach to String Figure-Making

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Abstract

String figure-making is a procedural activity carried out in many societies of oral tradition. It consists in producing geometrical forms using a string knotted into a loop. In 1988, an American mathematician, Thomas Storer (1938-2006), published a long article in which he developed several formal approaches of string figure-making. One of these is the Heart-sequence. Passing the string around a finger forms a "loop". The point is then to focus, during the process, on the movements of the loops without taking into account the way the fingers operate to make these loops, and to convert these movements into a mathematical formula. This mathematical approach to string figures allows a new reading of string figure procedures which makes it possible to classify them and is a promising way to carry out a comparative study of string figure corpora. This paper concentrates on string figure algorithms leading to "double-sided lozenge" final figures. Although various different methods in forming such a "double-sided lozenge" can be found in geographically and culturally distant societies, only two classes of heart-sequences will be identified.

Key Words: String figures, algorithm, modeling, classification, ethnomathematics.
1 Introduction

String figure-making consists in applying a succession of operations to a string (knotted into a loop), mostly using the fingers and sometimes the feet, the wrists or the mouth. This succession of operations, which is generally performed by one individual or sometimes by two, is intended to generate a final figure. For over a hundred years this practice has been observed by anthropologists in many regions of the world, especially within 'oral tradition' societies.

In a previous work, using concepts such as 'elementary operation', 'procedure', 'sub-procedure', 'iteration' and 'transformation', I discussed why and how string figures can be seen as the product of a mathematical activity. A string figure process can always be analyzed as a series of 'simple movements' that I call 'elementary operations' in the sense that the making of any string figure of a given corpus can be described by referring to a certain number of these operations—See below the elementary operations 'Picking up' a string (pictures 1a-b) and 'Twisting' a loop (pictures 1c-d).

Therefore a string figure can be seen as the result of a procedure consisting in a succession of elementary operations. The production of (new) string figure procedures can thus be regarded as the result of an intellectual process of organizing elementary operations through genuine
"algorithms". The analysis of numerous published collections of string figures, as well as the ethnographical data I have collected, provides evidence that many ordered sets of operations with a noticeable impact on certain configurations of the string have been clearly identified, memorized and sometimes named by string figure practitioners within different societies. Based on an algorithmic practice, the production of string figure algorithms is also of a "geometrical" and "topological" order in the sense that it is based on investigations into complex spatial configurations, aiming at displaying either a two-dimensional or a three-dimensional figure.

The practice of string figure-making is still very much alive in some societies. In the last few years, I carried out ethnographical research among the Trobriand Islanders of Papua New-Guinea and among the Guarani-Nandeva of the Chaco, Paraguay, collecting original corpora of string figures in both these areas. In these societies, many people know how to perform numerous string figures, and I often had the opportunity to work with real 'experts'. Nevertheless, I have never met anyone with the ability or even the desire to invent a new string figure. Therefore, I can only speculate about which methods were carried out by the actors to create new string figure algorithms. In order to hypothesize about these methods, I felt it necessary to better understand the impact of ordered sets of elementary operations on particular configurations of the string, with the objective to describe and analyze the phenomenon mathematically.

American mathematician Thomas Frederick Storer (1938-2006) published a long article in which he developed several different mathematical approaches on string figures (Storer, 1988). One of these, the 'Heart-sequence' of a string figure, has been of fundamental importance in my personal investigations. When the string is placed around a finger, it forms a "loop". The idea is then to focus on the movements of the loops without taking into account the way the fingers operate on them. By focusing on these movements during the process, and by converting them into a mathematical formula, the heart-sequence gives a "topological" view of a string figure algorithm.

In this article, I will first introduce Thomas Storer and his interest in string figure-making. Then, Storer's concept of Heart-sequence will be precisely described and commented. Finally, I will demonstrate its efficiency to analyze and compare different string figure algorithms leading to the same or very close final figures, that I call "double sided-lozenge string figures". The making of such a string figure, using different methods, was observed in a number of oral tradition societies. The heart-sequence conceptual tool allows the making of these figures to be compared at a "topological" level i.e. by comparing the movement of the string or loops that the elementary operations imply.
Storer’s interest in string figures

Thomas Storer spent most of his career at the University of Michigan, Ann Arbor, teaching mathematics and carrying out research in Combinatorics. Storer was a native American Navajo and he was known as one of the first native American to earn a Ph.D. degree in mathematics in the United States and to reach the position of full professor in a major university. He was also a string figure enthusiast, member of the International String Figure Association (ISFA) (Sherman, 2007). The goal of this organization, which was founded in 1978 by Japanese mathematician Hiroshi Noguchi and Anglican clergyman Philip Noble, is to bring together people of all nationalities, scholars and enthusiasts, to gather and distribute string figure knowledge. Nowadays, it has a hundred members, including some mathematicians and anthropologists, and publishes an annual review\(^1\).

Storer, who had learnt his first string figures in his childhood from his grandmother and friends, became well acquainted with the subject as shown by the following extract\(^2\): ... , in 1958 – having learned some 20 string-figures from my grandmother (all she knew!) and perhaps, another dozen from my friends while growing up – i discovered the little pamphlet by Rohrbough (ed.): FUN WITH FOLKLORE, with its two figures, Takapau and Brush House, taken from J.C. Andersen: MAORI STRING FIGURES. I could hardly believe my good fortune - that very educated and learned people had actually written about such things - and i devoured all the literature i could get my hands on. (Storer, 1988, p. iii)

Notice his surprise when he discovered that 'educated and learned people' had written about string figures. Storer did not say more about that. However it is clear that this literature intensified his interest in the subject. Storer strove to acquire a good knowledge of the bibliography on this topic. And so, in 1985, he published the first edition of String Figure Bibliography in

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\(^1\)See the website: www.isfa.org.
\(^2\)For unknown reason, Storer wrote 'i' instead of 'I' as the first personal pronoun.
the Bulletin of ISFA³. The need for a better understanding of the phenomenon grew along with his interest in string figures.

After learning my thousandth or so figure, I began searching for a book or article which spoke to the beautiful "system" which I dimly apprehended underlying these disparate string-figures - to no avail. The wordy ramblings of collectors were too imprecise to satisfy, and topological Knot-Theorists apparently dismissed the entirety of the string figures of the world as "trivial". And, although I learned a great deal from both groups of writers, I hungered for an approach which was neither too weak to be effective, nor so powerful that it identified (and as "trivial", at that) all the objects of my insatiable interest. And, since such work still does not exist, to my knowledge, I have decided to write one, chronicling my development of such a system over the ensuing years. (Storer, 1988, p. iii)

During his sabbatical year of 1988, Storer spent hours carrying out research on string figures⁴. This led him to publish a long paper entitled String-Figures in the Bulletin of the String Figure Association (Storer, 1988). As the article's foreword emphasizes, Storer was convinced of the existence of a "structure" underlying the set of all string figures. His project was clearly to devise formal tools in order to shed some light on this underlying "structure".

The purpose behind these researches is twofold: 1). To explicate the "structure" underlying the set of all string-figures by exploring their interrelations, and 2). "to conserve" string figures uniquely, in a manner not heretofore possible, through the development of an unambiguous formal language for their discussion (Storer, 1988, p. i).

To carry out this project, the mathematician developed different formal approaches on string figures. Storer created three conceptual tools which are presented in the first part of his paper under the title "Systemology". Then, in the next four parts, using these conceptual tools, Storer carried out a deep analysis of four string figures. For each of them, he explored their interrelations with other string figures. Working in this way, Storer introduced a methodology which could help us, in the long term, to "explicate the structure underlying the set of all string-figures." All of the Storer's formal approaches on string figure processes merit description and analysis, which I will carry out in later works. However, I will limit myself here to the description and the use of the concept of Heart-sequence.

³Two other editions followed in 1996 and 2000.
⁴This point is not mentioned in the article (Storer, 1988); it was mentioned by Storer's wife, Karen Storer, in her introduction of the article "Someone who Loved the String: A tribute to Tom Storer" (Sherman, 2007).
3  Storer’s Concept of Heart-Sequence

Before introducing this formal tool, I first need to define the notations used in this context.

3.1 Labeling the Fingers

Fingers are numbered from 1 (thumb) to 5 (little finger). $R$, $L$ and $B$ indicate "right hand", "left hand" and "both hands" respectively. In this way, $R1$ is the notation of the right thumb, whereas $L2$ denotes the left index.

The ten fingers are sometimes helped by the mouth ($M$), a big toe ($T$) or the wrist ($W$). All these are all termed "Functors" by the mathematician.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Thumb</td>
</tr>
<tr>
<td>2</td>
<td>Index</td>
</tr>
<tr>
<td>3</td>
<td>Middle finger</td>
</tr>
<tr>
<td>4</td>
<td>Ring finger</td>
</tr>
<tr>
<td>5</td>
<td>Little finger</td>
</tr>
<tr>
<td>$Ri$</td>
<td>$i^{th}$ finger of the right hand</td>
</tr>
<tr>
<td>$Li$</td>
<td>$i^{th}$ finger of the left hand</td>
</tr>
<tr>
<td>$R, L, B$</td>
<td>Right Hand, Left Hand, Both Hands</td>
</tr>
<tr>
<td>$M$</td>
<td>Mouth</td>
</tr>
<tr>
<td>$T$</td>
<td>Great toe</td>
</tr>
<tr>
<td>$W$</td>
<td>Wrist</td>
</tr>
</tbody>
</table>

3.2 Labeling the Objects

The Functors operate on what Storer calls "Objects". The Objects are separated into two groups: "strings" and "loops". A loop is formed when the string passes around a finger. Picture 3a shows a loop made on the left index $L2$. Storer denotes a loop by using the symbol $\infty$.

When $i \in \{1, 2, 3, 4, 5\}$, $Li \infty$ symbolizes the loop made on the $i^{th}$ finger of the left hand (for example, the loop on pictures 3a-b will be noted $L2\infty$). In the same way, $Ri \infty$ will define a loop made on the $i^{th}$ finger of the right hand. The "string" making the loop is divided into three parts. The one which lies on the "dorsal" side of the finger is written symbolically $Rid$ or
Lid for a loop made on the $i^{th}$ finger. The "near" string of a loop is the string closest to the practitioner. The other one is denoted as the "far" string (picture 3b).

3a- Loop formed on the left index

3b- $L_{2\infty}$ and its strings

The notations will be the following:
- $Rin$: right near string on the $i^{th}$ finger
- $Rif$: right far string on the $i^{th}$ finger
- $Lin$: left near string on the $i^{th}$ finger
- $Lif$: left near string on the $i^{th}$ finger

Anthropologists Alfred Cort Haddon and William H. R. Rivers defined "Position I" as the initial position obtained when loops are formed on the thumb and little finger of both hands\(^5\). In this case, the left and right palmar strings (string which lies on the palm of the hand) are then created (picture 4). These two palmar strings will be denoted as $Lp$ and $Rp$.

4- Position I

\(^5\)Cambridge anthropologist Alfred Cort Haddon (1855-1940), in collaboration with anthropologist, neurologist and psychiatrist, William H. R. Rivers (1864-1922), carried out the first significant study on string figures. In 1902, they published an article entitled 'A Method of Recording String Figures and Tricks' (Haddon & Rivers, 1902). In this paper the authors explained their methodology for collecting string figures and tricks. Moreover they proved the efficiency of their nomenclature by writing down the making of twelve Melanesian string figures that had been collected during the 1898-99 Cambridge expedition conducted by Haddon in the Torres Straits. The goal of the article was clearly to induce their colleagues to pay attention to the topic and help them to collect string figures in their own fields. For Haddon & Rivers’ definition of 'Position I’, see (Haddon & Rivers, 1902, p. 148).
As shown above, the string connecting \( L5f \) to \( R5f \) and the one between \( R1n \) to \( L1n \) will be noted simply \( 5f \) and \( 1n \) respectively.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Loops</td>
<td></td>
</tr>
<tr>
<td>( Li\infty )</td>
<td>Loop held by ( i^{th} ) finger of the left hand</td>
</tr>
<tr>
<td>( Ri\infty )</td>
<td>Loop held by ( i^{th} ) finger of the right hand</td>
</tr>
<tr>
<td>( W\infty )</td>
<td>Loop on the wrist</td>
</tr>
<tr>
<td>Strings</td>
<td></td>
</tr>
<tr>
<td>( Lif )</td>
<td>Far string of the loop held by the finger ( Li )</td>
</tr>
<tr>
<td>( Rin )</td>
<td>Near string of the loop held by the finger ( Ri )</td>
</tr>
<tr>
<td>( Lid )</td>
<td>Dorsal string of the loop held by the finger ( Li )</td>
</tr>
<tr>
<td>( if )</td>
<td>Entire string encompassing the connected ( Lif ) and ( Rif )</td>
</tr>
<tr>
<td>( in )</td>
<td>Entire string encompassing the connected ( Lin ) and ( Rin )</td>
</tr>
<tr>
<td>( Rp )</td>
<td>Right palmar string</td>
</tr>
<tr>
<td>( Lp )</td>
<td>Left palmar string</td>
</tr>
</tbody>
</table>

### 3.3 Some operations

We will now examine the coding of the elementary operations "Releasing" (a loop) and "Extending" (the string) that are used in the writing of Heart-sequences.

#### 3.3.1 Operation "Releasing"

When the \( i^{th} \) finger of the right or left hand carries a single loop, the operation of "releasing" this loop is symbolized by \( \square Ri\infty \) or \( \square Li\infty \).

As in picture 5a, graphically, I will indicate the release of a loop by drawing a full square on the corresponding finger.
When the specification left or right ($L$ or $R$) is omitted, it means that the same operation has to be done simultaneously on both hands. In such a way, $\Box i \infty$, indicates the release of both $R_i$ and $L_i$ single loops. Finally, $\Box i \infty$ will be often noted simply as $\Box i$.

### 3.3.2 Operation "Extending"

"|" indicates that the hands have to move apart in order to absorb the slack on the string.

![Image of hand configuration](image)

#### 6a- Extending

#### 6b- Done

### 3.3.3 Openings

An "Opening" is a sequence of elementary operations by which a string figure algorithm starts to create some loops on the fingers. For instance, the one called Opening A\(^6\) consists in starting from Position I, to form six loops (on the thumbs, indices and little fingers), as shown in pictures 7a-c.

![Image of hand configuration](image)

#### 7a- Position I

![Image of hand configuration](image)

#### 7b

![Image of hand configuration](image)

#### 7c- Opening A configuration

The openings are noted $O$ by Storer. Opening A is thus encoded $O.A$.

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\(^6\)As for "Position I", the expression "Opening A" is due to the Haddon and Rivers (1902). Opening A has a high occurrence in numerous corpora from many different geographical areas.
3.4 Heart-Sequence of a string figure: the example of Niu

Storer points out that string figures can be seen as the result of operations on “loops” leaving aside the way “functors” operate on them. In other words, if one had the opportunity to perform a string figure in the dark with a fluorescent string, the movements of the string observed without seeing the fingers could be summarized in a certain number of passages of a loop either around or through another loop. In order to give a formal description of the latter phenomenon, Storer has created a symbolic notation which formalizes the motion of loops during the procedure through a mathematical formula: the “Heart-sequence”.

3.4.1 The procedure Niu

As a first example, Storer refers to the string figure named Brokhos which could be, according to him, the oldest string figure known in Western literature.3

The oldest known string figure in western literature is attributed to the Greek Heraklas, about the first century A.D., in a manuscript entitled “Brokhos”. And, although the original work did not survive, it is extensively cited in the medical treatise “Iatrikon Synagogos” by one Oribasius of Pergamum.8

The string figure chosen by Storer is indeed very close to the Solomon string figure Niu collected by the anthropologist Raymond Firth in 1928 and published in 1978 by Honor Maude9 (Maude, 1978, p. 1). The latter procedure will be used below for comparative purposes in the

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3The reader will find interesting discussions about this hypothetical oldest description of a string figure in a paper by Lawrence G. Miller entitled “The Earliest(?) Description of a String Figure” (Miller, 1945), and in the article by Joseph D’Antoni entitled “Plinthios Brokhos, The Earliest Account of a string figure construction” (D’Antoni, 1997). In these papers, the authors interpret a description of a Brokhos (Greek word for bandage noose) called Plinthios. The latter is a knotting procedure leading to a rhomboidal shape. The discussion is based on the literal translation of Bussemaker and Daremberg’s French translation [Bussemaker & Daremberg, Oeuvre d’Oribase, Paris, 1862, 6 volumes] of the original Greek text by Orisabius, the oldest extant copy of which is the Laurentian Library MS. 74.7, sometimes called the Codex of Nicetas. Storer asserts that “we cannot know [for certain] Heraklas’ method of construction” and does not actually follow the interpretation of these authors. Therefore, Storer provides another construction process which leads to a similar rhomboidal design that he certainly thought more accurate for his discussion.

8T. Storer refers to C.L. Day, Quipus and Witches Knots, p. 124, where this figure appears as n°13, the ‘4-loop Plinthios Brokhos’ or ‘4-loop bandage noose’ (Storer, 1988, p. 49).

9Honor Maude (1905-2001) was Henry Evans Maude’s wife. The latter started his career as a civil servant in the British colonies in the Pacific and became professor of history of the Pacific at the Australian National University. During the Maudes’ stays in the Pacific Islands, Honor Maude became interested in string figures and made numerous collections throughout the Pacific.
fourth section of this article. Therefore, I will continue my discussion by writing down the heart-sequence of the string figure *Niu*.

I will not rewrite here the exact instructions given by Honor Maude. Instead, I will introduce a more formal description based on it and largely inspired by the terminology used by ISFA members. The idea is roughly to make the instructions more concise, using Storer’s notation of Functors and Objects.

**Step 1:** Opening A (picture 8a).

**Step 2:** Distally\(^{10}\), insert 1 into 2 loops. 1 picks up 2\(f\). 1 returns to position (picture 8b).

**Step 3:** Proximally, insert 3 into proximal 1 loops\(^{11}\). 3 picks up proximal 1\(f\) and return to position (picture 8c).

![Picture 8a](image1)

![Picture 8b](image2)

![Picture 8c](image3)

![Picture 8d](image4)

**Step 4:** Release 1 and extend (picture 8d).

**Step 5:** Distally, insert 1 into 2 loops. 1 passes proximal to 3 loops. Proximally, insert 1 into 5 loops, then pick up 5\(n\) and return (picture 8e).

**Step 6:** Release 5 and extend (picture 8f).

**Step 7:** Release 2 and extend (pictures 8g-h).

\(^{10}\)According to Haddon & Rivers’ nomenclature, the adjectives 'distal' and 'proximal' mean 'towards the tip of finger' and 'towards the wrist' respectively (Haddon & Rivers, 1902). Honor Maude used the expression 'distal to' and 'proximal to' as abbreviations of 'from distal side of' and 'from proximal side of' respectively. These two expressions later became the adverbs 'distally' and 'proximally'. These terms are still used by ISFA.
3.4.2 Analysis of steps 1 to 4

In the three steps above (2 to 4) taken together, it can be seen, by focusing on what happens to the previous thumb loops (1∞), that these loops pass from above through the index loops (2∞), and are transferred to the middle fingers. Pictures 9a-j show this passage with the movements of the right hand loops. We can distinctly see that the operations performed by $R_1$ and $R_3$ on the string causes the passage of $R_1$ loop (white one) from above through $R_2$ loop (black one).

\[\text{members. Here (when fingers are pointing up), "distally" is equivalent to "from above".}\]

\[\text{When a finger carries two loops, one is located towards the tip of the finger - the distal one - and the other one is close to the palm - the proximal one.}\]
The fourth step is the release of the thumbs. This entails that the white thumb loops \( (1\infty) \) are finally transferred to the middle fingers (pictures 9i-j). So, we observe that the previous \( R1 \) loop (white) has been passed from above through \( R2 \) loop (black) and that it is held by and is transferred to \( R3 \) at the end of the process. The motion of the white loop can be summarized by the diagrams in the following pictures 10a-b.

To convince the reader that such is the case, let us pass \( R3 \) loop (white) from below through \( R2 \) loop using \( R1 \) as shown in the following pictures 11a-e, and show that this is the inverse operation in the sense that it will take us back to the position that immediately follows Opening A.

The movement of passing \( R1\infty \) from above through \( R2\infty \) is noted: \( \overrightarrow{R1\infty} \downarrow (R2\infty) \). The arrow pointing right over the symbol \( R1\infty \) means that the loop \( R1\infty \) passes away from the practitioner and over all intermediate strings (none here). Moreover, the arrow pointing down indicates the insertion from above of \( R1\infty \) through \( R2\infty \).
More generally $X \rightarrow \infty \downarrow (Y \infty)$ will mean: pass the loop $X \infty$ away from you (over all intermediate strings - if any) - this action is symbolized by the top arrow pointing right - and insert them from above into the loop $Y \infty$ - action symbolized by the arrow $\downarrow$ pointing down.

In a similar way:

- $X \rightarrow \infty \uparrow (Y \infty)$ will mean: pass the loop $X \infty$ away from you (under all intermediate strings - if any) - this action is symbolized by the bottom arrow pointing right - and insert them from below into the loop $Y \infty$ - action symbolized by the arrow $\uparrow$ pointing up.

- $X \rightarrow \infty \uparrow (Y \infty)$ will mean: pass the loop $X \infty$ towards you (over all intermediate strings - if any) - this action is symbolized by the top arrow pointing left - and insert them from below into the loop $Y \infty$.

- $X \rightarrow \infty \uparrow (Y \infty)$ will mean: pass the loop $X \infty$ towards you (under all intermediate strings - if any) - this action is action symbolized by the top arrow pointing left - and insert them from below into the loop $Y \infty$.

We obviously have four other possible situations: $X \rightarrow \infty \uparrow (Y \infty)$, $X \rightarrow \infty \downarrow (Y \infty)$, $X \rightarrow \infty \downarrow (Y \infty)$, $X \rightarrow \infty \downarrow (Y \infty)$.

In steps 2 to 4 of Niu above, the same operations occur symmetrically on both hands. Therefore, $L1 \infty$ is also inserted from above through $L2 \infty$. In this situation, the two symmetrical insertions $R1 \infty \downarrow (R2 \infty)$ and $L1 \infty \downarrow (L2 \infty)$ will be simply noted by omitting both $L$ and $R$ as it is done in operation "Releasing a loop" (see above). The two simultaneous and symmetrical instructions are then encoded as the single formula $\rightarrow 1 \infty \downarrow (2 \infty)$. 
In order to indicate that both thumb loops $1\infty$ are finally transferred to the middle fingers, Storer uses the notation $1\infty \rightarrow 3$ which is defined as follows: 'pass $1\infty$ away from you (under all intermediate strings - if any) and place it, as loops, directly upon 3'. The arrow pointing right under the symbol $1\infty$ is chosen here since just after the insertion through $2\infty$ (black), $1\infty$ (white) has to pass under the far index strings $2f$ before being transferred to the middle fingers. Finally, the colon ':' is used to punctuate the operations on loops. So, focusing on the motion of $1\infty$ the four first steps of Niu can be summarized as\textsuperscript{12}:

\[
1\infty \downarrow (2\infty) : 1\infty \rightarrow 3
\]

### 3.4.3 The three next steps of Niu

With the same point of view, the fifth and the sixth steps show a displacement of little finger loops $5\infty$ (dotted line)(pictures 12a-h). We see that the loops held by the little fingers ($5\infty$) pass towards the practitioner under all intermediate strings ($3f$ and $3n$), and go from below through the index loops $2\infty$ (black). This is encoded: $5\infty \uparrow (2\infty)$.

After this passage, the loops $5\infty$ (dotted line) are finally transferred to the thumbs. This transfer is noted $5\infty \rightarrow 1$ which means: 'pass $5\infty$ towards you over all intermediate strings (the string $2n$ in this case) and place it, as a loop, on 1.'

Finally, focusing on the motion of $5\infty$, the steps 5 and 6 of Niu can be summarized as:

\[5\infty \uparrow (2\infty) : 5\infty \rightarrow 1\]
3.4.4 Last step of Niu

At this point, in order to reach the final figure, the indices are released (□2) and the figure is extended gently (Step 7). This is coded: □2 |.

The Heart-sequence of the procedure Niu is then given by the following formula:

\[ \begin{align*}
Q.A : \left\{ \begin{array}{l}
\overrightarrow{1\infty} \downarrow (2\infty) : 1\infty \rightarrow 3 \\
\overleftarrow{5\infty} \uparrow (2\infty) : 5\infty \rightarrow 1
\end{array} \right\} : \square 2 |
\end{align*} \]

A heart-sequence always begins with an Opening (Q.A in the case of Niu) whose aim is to create a certain number of loops. It is only after the Opening that the analysis of the loops' movements can be written. Storer chose a presentation in columns to indicate that the succession of operations whose heart-sequences are \( \overrightarrow{1\infty} \downarrow (2\infty) : 1\infty \rightarrow 3 \) and \( \overleftarrow{5\infty} \uparrow (2\infty) : 5\infty \rightarrow 1 \) could be performed simultaneously. Of course, in practice, it is difficult to do so. In other words, one can visualize theoretically these simultaneous movements of loops without being able (in general) to perform them with fingers.

3.5 A few remarks before going further

Writing down the heart-sequence of a string figure implies a loss of information. This loss can be of two kinds. The first is obviously generated by the focus on the object "loop", causing a loss of information about the way the "functors" (fingers, feet, mouth) operate. The second concerns the order in which the movements of loops follow one another during the course of the algorithm. Following the order of the elementary operations of the procedure Niu, one should write down its heart-sequence as follows:

\[ \begin{align*}
Q.A : \overrightarrow{1\infty} \downarrow (2\infty) : 1\infty \rightarrow 3 : \overrightarrow{1\infty} \downarrow (2\infty) : 1\infty \rightarrow 3 : \square 2 |
\end{align*} \]

This sequence makes explicit that the movement of 1∞ occurs first, before the movement of 5∞. Even though a simultaneous sequence can be written in the case of Niu, we will see that in many other cases the order of displacements of the loops has to be taken into account to write down the accurate heart-sequence of a given string figure algorithm.
It is noticeable that different string figure algorithms can share exactly the same Heart-sequence. For instance, it is possible to find a procedure which differs from Niu’s one, but has a identical heart-sequence. Starting from Opening A, one can easily find out a procedure whose effect is, firstly, to insert $5\infty$ from below into $2\infty$ and, secondly, to insert $1\infty$ from above into $2\infty$.

Storer introduced the Heart-sequence concept but seldom used it. Although there is no evidence, I believe that the main reason for this lies in the difficulty of justifying the writing down of the heart-sequence of a given string figure procedure. As seen above, the use of pictures showing the movements of colored loops enables clear justifications to be given. Such a methodology would have been difficult to use two decades ago, before the development of certain digital devices. However, Storer was convinced that the Heart-sequence concept could be essential to study string figures:

We view the concept of "heart-sequence" as a Gedanken experiment of fundamental importance in the deeper understanding of the string figures of the world (Storer, 1988, p. 35).

3.6 Heart-sequence versus music score

As previously mentioned, two different string figure algorithms can share the same heart-sequence. At the same time, when a heart-sequence is given, it is theoretically possible to reconstruct a string figure algorithm, even though the solution is not unique, and sometimes the succession of elementary operations is not easy to find out (Vandendriessche, 2010, p. 231-246).

By analogy with the practice of a musical instrument, a heart-sequence can be seen as a music score. The reconstruction of a corresponding string figure algorithm would thus be analogous to the search of an accurate 'fingering' to 'play the music' with the instrument i.e. to implement a given heart-sequence with our body. Several fingerings are then possible. A string figure algorithm could thus be seen as the result of the connection between a 'heart-sequence' and a 'fingering' to implement it. I call 'basic fingering' the method which consists to implement a given heart-sequence in manipulating the loops in one hand by using the opposite one. For instance, to implement the sub-sequence $\overrightarrow{1\infty} \downarrow (2\infty) : 1\infty \rightarrow 3$ that we found within Niu, one can grasp $R1\infty$ (black) with the left hand (pictures 13a-c), then insert it from above.

13 For instance, it is possible to implement the operations on loops of one hand, using the opposite one. See what I define as "basic fingering" in the next Section 3.6.
into $R2\infty$ (white - pictures 13c-d), and place it on the middle finger (pictures 13d-f). Finally, the same operations can be repeated on the opposite side.

As far as I know, this kind of fingering rarely occurs in practice while making string figures. However, the basic fingering has been a useful tool to validate heart-sequences. Working in this way, one has to first find the heart-sequence of a procedure in the original algorithm. Afterwards, the formula which has been worked out can be validated, step by step, implementing it through a basic fingering.

4 Heart-sequences and "looking alike" String Figures

I define the "drawing" of a string figure as the geometric design that can be extracted from it without taking into account the exact path of the string. I will say that two final figures "look alike" if they show the same "drawing", even though they differ on some (or all) of their 'simple crossings'. For instance, in that sense, the two final figures in the pictures 14a-b "look alike".

If two final figures $A$ and $B$ look alike, I will say that the figure $A$ is a $B$ "lookalike". The concept of Heart-sequence is most efficient to analyze and classify a set of string figure algorithms leading to 'looking alike' final figures. Many procedures starting with Opening A and forming a figure looking like the final figure of Niu can be found in ethnographical literature. Also, I have personally collected some of them. I will call this set of procedures the "double-sided lozenge" family.
4.1 Kapiwa from the Trobriand Islands

In the Summer of 2006, in the Trobriand Islands (Papua New Guinea), I collected a string figure procedure, named *Kapiwa* (bee), leading to a 'double-sided lozenge'. Let us determine the heart-sequence and compare it to *Niu*’s heart-sequence.

4.1.1 The procedure *Kapiwa* and its Heart-sequence

Consider the first operations of *Kapiwa*. The hands operate symmetrically and the pictures below show the moves of the right hand. *Kapiwa* starts with Opening A (Step 1). Then, two steps taken together allow to transfer the thumb loops to the wrists (pictures 15a-k).

Step 2: Distally, insert 2345 into 1 loops. 2345 grasp 1f, 2 loops and 5 loops. Pass 1f to the dorsal side of the hands, while releasing 1 then, place hands facing each other (pictures 15a-h).

Step 3: 1 picks up lower 2n in order to transfer the former 1 loops to the wrist (pictures 15h-k).
This is how, throughout several steps, thumb loops \(1 \infty\) (white) are rotated 180° anticlockwise (for an observer located on the left side of the practitioner), and are transferred to the wrists.

When a loop held by a Functor \(F\) is rotated 180° anticlockwise 'for an observer located on the left side of the practitioner', Storer notes: \(> F \infty \ (< F \infty,\ if\ the\ rotation\ is\ performed\ clockwise.)\)

The movements of \(1 \infty\) mentioned above can then be summarized by the sequence: \(Q.A : <1 \infty : \overrightarrow{1 \infty} \rightarrow w,\) that I will note simply as \(Q.A : <1 \infty \rightarrow w.\)

By disregarding the hands, the following diagram demonstrates that the operations described above lead to a configuration equivalent to the one obtained by rotating thumb loops \(1 \infty\) anticlockwise of 180° (for an observer located on the left side of the practitioner) and by placing them again on their original fingers.
The point is actually to rotate thumb loops and transfer them to the wrist in order to allow the thumbs to operate freely. The procedure continues through the following step:

Step 4: Pass 1 proximal to all intermediate strings. Proximally, insert 1 into 5 loops. Pick up 5f and return to position. Release 5 (pictures 16a-h).

The effect of this succession of operations is to pass little finger loops 5∞ (black) under wrist loops w∞ (white). Also, during this movement, 5∞ are rotated 360° clockwise. Then, the latter loops are transferred temporarily to the thumbs before continuing their movement. This is symbolized: \( \gg \frac{5\infty}{w\infty} \).
From this stage, Step 5 consists in transferring the index loops $2\infty$ to the little fingers (pictures 16i-l). The procedure continues through the following two steps:

Step 6: Proximally, insert 1 into 5 loop. 1 picks up $5n$ and return to position (pictures 16m-o).

Step 7: 2 picks up proximal 1f, Then 1 presses against the side of 2 to trap the string that runs from 1 to 2 and the string that runs from 1 to 5. Finally, the wrists begin to rotate (pictures 16p-r).

One can see in pictures 16m-r that the original little finger loops $5\infty$ (black), now held by the thumbs, pass over the wrist loops $w\infty$ (white), then, from below, through the original index loops $2\infty$ (dotted line), now held by the little fingers.

This will be encoded: $\overrightarrow{5\infty}$ ($w\infty$) : $\overrightarrow{5\infty} \uparrow (2\infty)$.

The procedure ends like this:

Step 8: Release 5. Release 1 while rotating the hands, turning the palms away (pictures 16s-v)$^{14}$.

So, at the end of the process the original index loops $2\infty$ (dotted line) are released and the original little finger loops $5\infty$ (black) are transferred to the indices. This can be written: $\square 2 : \overrightarrow{5\infty} \longrightarrow 2$.

---

$^{14}$See also the procedure Kapiwa in the Annex II.
Finally, the heart-sequence of *Kapiwa* is given by

\[ Q.A : \left\langle \frac{\infty}{1} \to w : >> \right\rangle \left( w \infty \right) : \left( \frac{5}{\infty} \right) \quad \left( w \infty \right) : \left( \frac{5}{\infty} \right) \uparrow \left( 2 \infty \right) : \left[ 2 : \left( \frac{5}{\infty} \right) \to 2 \right] \]

4.1.2 Comparison with *Niu*

**Heart-sequence comparison** The movement of loops involved in the making of the string figure *Kapiwa* is different from the one in *Niu*. The heart-sequence above reveals that the movement of loops involved in the making of the string figure *Kapiwa* is different from the one in *Niu*. The heart-sequence above reveals that the
string figure algorithm *Kapiwa* entails the movement of the single pair of little finger loops $5\infty$, passing around wrist loops $w\infty$ (former $1\infty$), and inserting from below into index loops $2\infty$. By contrast, the heart-sequence of *Niu* is based on the movement of two pairs of loops, thumb loops $1\infty$ and little finger loops $5\infty$, both inserting into index loops $2\infty$, one from above and the other from below.

Heart-sequence of *Kapiwa*:

$$Q.A : \vec{1}\infty (5\infty) : > \frac{1}{2}\infty (5\infty) : \frac{1}{2}\infty (2\infty) : \boxed{2} : > \frac{1}{2}\infty \rightarrow 2 |$$

Heart-sequence of *Niu*:

$$Q.A : \left\{\begin{array}{c}
\vec{1}\infty (2\infty) : \vec{1}\infty \rightarrow 3 \\
\frac{5}{2}\infty (2\infty) : \frac{5}{2}\infty \rightarrow 1
\end{array}\right\} : \boxed{2} |$$

**Final figures comparison**  Let us focus on the final figures of *Niu* and *Kapiwa* procedures. In order to allow the final figures to be compared, we have to lay out the figures each time in the same way. I have chosen the practitioner’s viewpoint. Working in this way, it is noteworthy that the two final figures *Niu* and *Kapiwa* are absolutely identical. As seen above, the heart-sequence of *Kapiwa* is clearly different from procedure *Niu*’s one. We have thus identified two different string figure algorithms whose respective heart-sequences are definitely different, although they lead to the same final figure *i.e* the same ’Knots’ (crossings included).

Let us turn to a third example. In October 2005, I learnt the following procedure, called *Jasytata* (stars), among the Guarani-Ñandeva from the Chaco, Paraguay. This procedure also leads to a double-sided lozenge, which is not identical to the previous one made through *Niu* or *Kapiwa*. However, there exists a symmetry relationship between *Jasytata* and *Kapiwa* final figures which can be analyzed through the comparison of their heart-sequences.

### 4.2 *Jasytata* from Chaco, Paraguay

#### 4.2.1 The procedure *Jasytata* and its Heart-sequence

The procedure starts with Opening A (Step 1 - picture 17a). Then, the hands operate one after the other. Pictures 17b-j show the left hand manipulating the loops on the right hand. The second step can be described as follows:

Step 2: Distally insert $L2$ into $R2$ loop. Pass $L2$ away from you distal to $R5$ loop, then towards you proximal to $R5$ loop. Pass $L2$ towards you distal to $R1$ loop, then pick up both $R1f$ and $R1n$. $L2$ return to position (picture 17b). Seize both $R1n$ and $R1f$ between $L2$ and...
L3. Release $R_1$ (picture 17c-e). Distally, insert $R_1$ into the loop seized between $L_2$ and $L_3$. Transfer this loop to $R_1$. Extend (picture 17f-j).
The aim of these operations (pictures 17a-j) is actually to make $R1\infty$ (black) turn around $R5\infty$ (white), passing under (away from you) and then above (towards you) $R5\infty$ (dotted line). Finally, $R1\infty$ (black) is inserted from below into $R2\infty$ (dotted line). During this movement $R1\infty$ is rotated 360° clockwise: 180° while $L2$ picks up $R1\infty$ (pictures 17a-f), and 180° while the original loop $R1\infty$ - grasped by $L2$ and $L3$ - is transferred to $R1$ (pictures 17g-j above). This can be formalized:

$$Q.A : R1\infty (R5\infty) : \leftarrow \arrow{R1\infty (R5\infty)} : \lessgtr R1\infty \uparrow (R2\infty).$$

The same operations are then applied on the left hand. Although it is impossible in practice, we can consider that the movement of $R1\infty$ and $L1\infty$ may theoretically happen simultaneously. So, the heart-sequence will be simply written as follows:

$$Q.A : 1\infty (5\infty) : \leftarrow 1\infty (5\infty) : \lessgtr 1\infty \uparrow (2\infty).$$

At this stage, the indices are released and the string is extended, formally written $\Box 2|$. This leads to a "double-sided lozenge" final figure (pictures 17k-l).15

![17k](image17k.png) ![17l](image17l.png)

The heart-sequence of *Jasytata* is then given by the following formula:

$$Q.A : 1\infty (5\infty) : \leftarrow 1\infty (5\infty) : \lessgtr 1\infty \uparrow (2\infty) : \Box 2|.$$

As for *Kapiwa*, it is the movement of a single pair of loops ($1\infty$, in this case), passing around another ones ($5\infty$), finally inserted from below into index loops $2\infty$.

### 4.2.2 Comparison of *Kapiwa* and *Jasytata*

Let us consider the following two sub-sequences $X$ and $Y$ of the heart-sequences of *Kapiwa* and *Jasytata* respectively.

---

15See also the procedure *Jasytata* in the Annex II.
Let us now focus on the sub-sequence $X$ within $Kapiwa$’s heart-sequence:

$$X = \overrightarrow{5\infty}(w\infty) : \overleftarrow{5\infty}(w\infty) : >> \overrightarrow{5\infty}(2\infty) : \square 2 : \overrightarrow{5\infty} \rightarrow 2 |(Kapiwa)$$

Wrist loops $w\infty$ were originally held by the thumbs. Remember that the point was to transfer thumb loops to the wrist in order to let the thumbs operate freely. Therefore, we can substitute $1\infty$ to $w\infty$ without changing the "spirit" of the sub-sequence $X$.

Thus, sub-sequence $X \iff \overleftarrow{5\infty}(1\infty) : \overrightarrow{5\infty}(1\infty) : >> \overrightarrow{5\infty}(2\infty) : \square 2 |(Jasytata)$

It is now easy to compare the sub-sequences $X$ and $Y$.

Sub-sequence $Y$: $\overrightarrow{1\infty}(5\infty) : \overleftarrow{1\infty}(5\infty) : <<< \overrightarrow{1\infty}(2\infty)$

Sub-sequence $X \iff \overleftarrow{5\infty}(1\infty) : \overrightarrow{5\infty}(1\infty) : >> \overrightarrow{5\infty}(2\infty)$

The comparison of these sub-sequences clearly reveals that the moves of the loops involved within the sub-sequence $X$ are the mirror moves of the loops within $Y$. This implies that $Jasytata$’s final figure should be the reflection of $Kapiwa$’s one with respect to a plane perpendicular to the figures’ plane. Actually, it is not exactly the case. This is due to the way by which the final figures are presented. When performing $Kapiwa$, the loops of the wrist come from the loops of the thumbs. If the thumbs had kept these loops, they would have pointed down, in order to present the final figure in a similar fashion. On the other hand, $Jasytata$’s final figure is presented with thumbs pointing up. Therefore, if we lay out the figures (as we have previously mentioned, projection in a plane keeping the practitioner’s viewpoint) and reverse (Reversal $R$ as defined in the diagram below) $Kapiwa$’s final figure before looking at the final figure in a mirror (Reflection $S$), we can see that the latter image is identical to $Jasytata$’s final figure.
By comparing the final figures of Kapiwa and Jasytata, one can see that all the crossings are reversed from one figure to the other. This implies that the final figure of Jasytata is the image of the final figure of Kapiwa under a reflection with respect to a plane parallel to the figure’s plane. Furthermore, one can verify that the composite transformation SoR is equivalent to such a reflection.

4.3 Classification

I call Group I the subset of the "double-sided lozenge family". It regroups those string figures whose heart-sequences begin with O.A and describe the movement of one pair of loops (generally 1∞ or 5∞), passing around a second (generally and respectively 5∞ or 1∞) and through a third.
pair of loops (generally $2\infty$). In such a way the procedures *Kapiwa* and *Jasytata* I described above belongs to Group I. Group II will be defined as the subset of the double-sided lozenge procedures. They also start with $O.A$, but their heart-sequences describe the movement of two pairs of loops (generally $1\infty$ and $5\infty$), both passing through a third pair of loops (generally $2\infty$), one from above and the other from below. In such a way, *Niu* is a member of Group II.

In a noteworthy way, every "double-sided lozenge" string figure, that I have learnt, either in the field or in anthropological papers, can be classified into either Group I or Group II. Table I in Annex I presents those from Oceania. As seen with the example of *Jasytata*, double-sided lozenge string figures can also be found in collections from South America (Chaco), as well as in others from Central Africa (Zande, West shore of Tanganika), and India (Gujarat). Table II in Annex I gives them according to Group I and II criteria.

As far as I can see, the heart-sequence of a string figure algorithm belonging to Group I is one of the four heart-sequences, "modulo" some transfers of loops, obtained from *Jasytata*’s heart-sequence under the action of a Klein group. This group is composed of the two reflections $S_1$ and $S_2$ with respect to the perpendicular planes $P_1$ and $P_2$ (picture 18), the symmetry with respect to the line $d = P_1 \cap P_2$ and the Identity $Id$ of the three dimensional space.

In the context of Opening A, the 'core' of these four heart-sequences are given by

- $Y \equiv 1\infty (5\infty) : \overleftarrow{1\infty (5\infty)} : \overleftarrow{1\infty \uparrow (2\infty)}$ (*Jasytata*)
- $S_1(Y) \equiv 5\infty (1\infty) : 5\overrightarrow{\infty (1\infty)} : 5\overrightarrow{\infty \uparrow (2\infty)}$ (*Kapiwa*)
- $S_2(Y) \equiv \overrightarrow{1\infty (5\infty)} : \overrightarrow{1\infty (5\infty)} : \overrightarrow{1\infty \downarrow (2\infty)}$
- $S_d(Y) \equiv \overleftarrow{5\infty (1\infty)} : \overleftarrow{5\infty (1\infty)} : \overleftarrow{5\infty \downarrow (2\infty)}$
These sequences can be experimented by using an accurate "basic fingering". The reader will thus verify that the release of the index at the end of the process makes a double-sided lozenge appear.

Remember that the core of the heart-sequence of Niu is

\[
\begin{align*}
\begin{cases}
\overline{1\infty} \downarrow (2\infty) & : \overline{1\infty} \rightarrow 3 \\
\overline{5\infty} \uparrow (2\infty) & : \overline{5\infty} \rightarrow 1
\end{cases}
\end{align*}
\]

Let us substitute the transfer \(\overline{1\infty} \rightarrow 3\) by \(\overline{1\infty} \rightarrow 5\).

\[
\begin{align*}
N : \begin{cases}
\overline{1\infty} \downarrow (2\infty) & : \overline{1\infty} \rightarrow 5 \\
\overline{5\infty} \uparrow (2\infty) & : \overline{5\infty} \rightarrow 1
\end{cases}
\end{align*}
\]

It is then easy to determine the heart-sequences obtained through the action of the Klein group. We have:

\[
S_2(N) : \begin{cases}
\overline{1\infty} \uparrow (2\infty) & : \overline{1\infty} \rightarrow 5 \\
\overline{5\infty} \downarrow (2\infty) & : \overline{5\infty} \rightarrow 1
\end{cases} \quad S_1(N) : \begin{cases}
\overline{5\infty} \downarrow (2\infty) & : \overline{5\infty} \rightarrow 1 \\
\overline{1\infty} \uparrow (2\infty) & : \overline{1\infty} \rightarrow 5
\end{cases}
\]

and,

\[
S_d(N) : \begin{cases}
\overline{5\infty} \uparrow (2\infty) & : \overline{5\infty} \rightarrow 1 \\
\overline{1\infty} \downarrow (2\infty) & : \overline{1\infty} \rightarrow 5
\end{cases}
\]

According to the fact that the two sub-sequences \(\overline{1\infty} \downarrow (2\infty) : \overline{1\infty} \rightarrow 5\) and \(\overline{5\infty} \uparrow (2\infty) : \overline{5\infty} \rightarrow 1\) within \(N\) can be interchanged or performed simultaneously (in theory), we have: \(S_2(N) \equiv S_1(N)\) and \(S_d(N) \equiv N\).

I define the procedures belonging to Group I (resp. Group II) as 'dynamically equivalent' in the sense that their heart-sequences consist in movements of loops which are related to one another under a plane or mirror symmetry. As shown above, when comparing Jasytata and Kapiwa, these symmetries (of the loops’ movement) clearly show the transformations that connect different final figures. Many of these 'dynamically' equivalent procedures probably emerged independently in different communities. Such mathematical activity—which would have consisted in working out algorithms leading to a double-sided lozenge—led to procedures that were similar in substance, but with a different form in geographically and culturally distant areas.

Tables 6.1 and 6.2 in Annex I show that some corpora of string figures contain one that is common to both groups (I and II). Such a phenomenon occurs in the Chaco (Jasytata and Estrella), on Nauru Island (Ekwan III, Eongatubabo), on the Tuamotus (Na tifai I, Na tifai II) and on the Solomon Islands (Niu, Nepe). The latter example is particularly interesting. Both
string figures Niu and Nepe (almost similar to Kapiwa) were recorded by Raymond Firth in the same small area (Reef Islands, Solomon)\textsuperscript{16} in 1928/29 (Maude, 1978, p. 1). Moreover, the final figures of these two procedures are entirely identical (crossings included). This seems to indicate that some practitioners or creators of string figures worked out two different procedures, based on different heart-sequences, to obtain a "double-sided lozenge". This brings to light the interest that some practitioners had in the procedures (heart-sequence + fingering): if they were only interested in the final figures, they probably would not have tried to find out different procedures to display identical string figures.

**Beyond the context of Opening A** The previous classification of the double-sided lozenge string figures has been carried out in the context of "Opening A". Many different openings can be found in the various corpora of string figures. However, the goal of these openings is always to obtain the first "stable" configuration, which consists in a taut state of the string with a certain number of loops created on the fingers. It is from these configurations that the movements of the loops can be analyzed, the heart-sequences written, and a comparative analysis carried out in the context of a particular opening. Furthermore, the heart-sequence concept can also be an efficient tool to compare the various first configurations obtained through different openings, and thus to compare string figure algorithms which do not start with the same opening. To illustrate that point, let us focus on the normal position obtained through the opening that I note $O.N$ (Opening N). It often occurs within the Chaco string figure procedures and has been first observed by anthropologists among the Navajo American Indians (pictures 19a-e).

Formally, the configuration in picture 19e can be reached from the configuration obtained through $O.A$ by releasing $5\infty$ and reversing both $2\infty$ and $1\infty$ clockwise according to the following sequence:

$$Conf(O.N) \equiv Conf(O.A) : \Box 5 : \left\{ \begin{array}{l} < 2\infty \\ < 1\infty \end{array} \right\}$$

where $Conf(O.N)$ and $Conf(O.A)$ designate the configuration obtained through $O.A$ and $O.N$ respectively.

I have shown elsewhere that sometimes the practitioners could have reached a configuration such as $Conf(O.N)$ from $Conf(O.A)$, and then worked out a more direct way to get it, thus creating a new Opening (Vandendriessche, 2010, p. 448-452).

\textsuperscript{16}The Reef Islands are a group of sixteen small coral Islands located 80 km from Santa Cruz Island in the eastern Solomon.
A few examples mentioned in anthropological papers suggest that the view of a string figure algorithm through its heart-sequence is sometimes a conceptualization made by the practitioners themselves. In his work on the Arviligjuarmiut string figures, missionary and ethnographer Guy Mary-Rousselière explains that the expression Anitidlugo means "to pass one (loop) into another" (Mary-Rousseliere, 1969, p. 5). Although he does not mention it, the use of parentheses seems to indicate that the word "loop" was implicit in the context of string figure-making. Unfortunately, Mary-Rousselière does not comment further on this term. Nevertheless, we may reasonably think that Anitidlugo referred to a succession of operations, often described in the Arctic string figure corpora, whose goal is clearly to pass one loop through another. In particular, French explorer Paul-Emile Victor describes it in his paper on the string figures of Ammassalik, Greenland (Victor, 1940) (Vandendriessche, 2007, p. 47). This example seems to indicate that some creators or practitioners from the Arctic would have identified the operation "Passing one (loop) into another" as central in the making of string figures. However, we cannot be sure that they saw the whole process through this viewpoint.

A hint is also given on the way the loops on one of the hands are sometimes grasped and manipulated by the fingers of the other hand in the making of some string figures. Jasytata, analyzed in this article, provides such an example. In fact, the "fingering" of Jasytata is not so far from what I have termed "basic fingering": the loops 1∞ are grasped and directly manipulated by the index and middle finger of the opposite hands (pictures 17a-h). This seems to indicate

5 Conclusion

A few examples mentioned in anthropological papers suggest that the view of a string figure algorithm through its heart-sequence is sometimes a conceptualization made by the practitioners themselves. In his work on the Arviligjuarmiut string figures, missionary and ethnographer Guy Mary-Rousselière explains that the expression Anitidlugo means "to pass one (loop) into another" (Mary-Rousseliere, 1969, p. 5). Although he does not mention it, the use of parentheses seems to indicate that the word "loop" was implicit in the context of string figure-making. Unfortunately, Mary-Rousselière does not comment further on this term. Nevertheless, we may reasonably think that Anitidlugo referred to a succession of operations, often described in the Arctic string figure corpora, whose goal is clearly to pass one loop through another. In particular, French explorer Paul-Emile Victor describes it in his paper on the string figures of Ammassalik, Greenland (Victor, 1940) (Vandendriessche, 2007, p. 47). This example seems to indicate that some creators or practitioners from the Arctic would have identified the operation "Passing one (loop) into another" as central in the making of string figures. However, we cannot be sure that they saw the whole process through this viewpoint.

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that the procedure *Jasytata* has been created in relation to an operative practice based on the movement of loops.

In this paper, we have seen that writing down a Heart-sequence consists in rewriting a string figure procedure as a new algorithm, which formalizes the movement of loops, ignoring the way fingers operate on them during this procedure. This is guided by the identification of either the sequences of operations on loops which can be theoretically carried out simultaneously (and thus in any order) or the ones which cannot be switched around.

Different string figure procedures can share exactly the same Heart-sequence. When it is not the case, their heart-sequences can be sometimes defined as 'equivalent'. We have seen that a few transfers of loops from one finger to another do not generally alter the 'spirit' of a given heart-sequence. Two heart-sequences can thus sometimes be seen as equivalent 'modulo' some transfers of loops. I have also suggested considering as 'dynamically equivalent' string figure procedures whose heart-sequences can be obtained from one another through symmetries, thus explaining certain symmetry relationship between final figures. These equivalences between Heart-sequences enable us to find a similarity between string figure procedures that are very different at first sight, thus providing a methodology to classify them.

Storer’s heart-sequence concept is relevant in every corpus of string figures that I have studied so far, and it provides a reliable and consistent tool to analyze string figure procedures as 'Observers'. We can thus hypothesize that the analysis of a large number of string figure procedures through their heart-sequence enables them to be classified. Heart-sequences allow what I suggested calling a 'topological' view on string figure procedures, allowing a better understanding of the elementary operations’ impact on the string, but also of the creation of particular patterns (such as the 'double-sided lozenge'). On the basis of this observer’s viewpoint, one can speculate on how the actors from different societies have explored these string figure procedures. It will be then crucial to compare and contrast the outcomes of this formal approach with the practitioners’ viewpoint within these societies.

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References


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## 6 Annex I: Classification

### 6.1 Oceania

<table>
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<tr>
<th>Names</th>
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<th>Group</th>
<th>Comments about the procedure</th>
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<td>Kapiwa</td>
<td>Trobriand Islands</td>
<td>I</td>
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<td>Solomon Islands</td>
<td>II</td>
<td>(Maude, 1978, p. 1-2)</td>
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<tr>
<td>Nepe (Moon)</td>
<td>Solomon Islands</td>
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<td>Close to Kapiwa (different beginning)</td>
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<td>Marquesas</td>
<td>I</td>
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<td>New Caledonia</td>
<td>I</td>
<td>Identical to Kapiwa (Compton, 1919, p. 214-215)</td>
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<td>Gilbert Islands</td>
<td>I</td>
<td>Close to Kapiwa (different beginning)</td>
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<td>Tuamotus, French Polynesia</td>
<td>II</td>
<td>Close to Niu (tiny difference at the end)</td>
</tr>
<tr>
<td>Na Tifai II</td>
<td>Tuamotus</td>
<td>I</td>
<td>Identical to Pu Kava (Maude &amp; Emory, 1979, p. 2)</td>
</tr>
<tr>
<td>Na Tifai III</td>
<td>Tuamotus</td>
<td>II</td>
<td>Identical to “Nameless” from New Caledonia, (Maue &amp; Emory, 1979, p. 3-4)</td>
</tr>
<tr>
<td>Foi nupu Pu Taumako</td>
<td>Tikopia Island</td>
<td>II</td>
<td>Close to Niu (different beginning) (Maude &amp; Firth, 1970, p. 18)</td>
</tr>
<tr>
<td>Wahine (Butterfly)</td>
<td>New-Zealand</td>
<td>I</td>
<td>Identical to Pu kava (Andersen, 1927, p. 242)</td>
</tr>
<tr>
<td>Ekwan II (Sun)</td>
<td>Nauru Island</td>
<td>I</td>
<td>(Maude, 1971, p. 62-63)</td>
</tr>
<tr>
<td>Ekwan III (Sun)</td>
<td>Nauru Island</td>
<td>I</td>
<td>Identical to Pu kava (with a Caroline Extension at the end) (Maude, 1971, p. 63-64)</td>
</tr>
<tr>
<td>Eongatubabo</td>
<td>Nauru Island</td>
<td>II</td>
<td>(Maude, 1971, p. 77) Steps 1-5 identical to Niu, then continuation by iteration.</td>
</tr>
<tr>
<td>Paa (Crab)</td>
<td>Samoa</td>
<td>II</td>
<td>Identical to Niu (Hornell, 1927, p. 73-74)</td>
</tr>
<tr>
<td>Sasa (White Cockatoo)</td>
<td>Numba Village, Managalas and Musa, PNG</td>
<td>I</td>
<td>Identical to Nepe (Noble, 1979, p. 35-36)</td>
</tr>
</tbody>
</table>
6.2 Elsewhere

<table>
<thead>
<tr>
<th>Names</th>
<th>From</th>
<th>Group</th>
<th>Comments about the procedure</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jasytata (Star)</td>
<td>Chaco, Paraguay</td>
<td>I</td>
<td>Collected by Eric Vandendriessche (2010)</td>
</tr>
<tr>
<td>Estrella (Star)</td>
<td>Chaco, Paraguay</td>
<td>II</td>
<td>Collected by Eric Vandendriessche (2010)</td>
</tr>
<tr>
<td>Mwezi (Moon)</td>
<td>Murungu, West Tanganika, Central Africa</td>
<td>I</td>
<td>Mirror moves of Pu Kava (Cunnington, 1906, p. 129)</td>
</tr>
<tr>
<td>Kumba ma De (Male and Female)</td>
<td>Zande People, Central Africa</td>
<td>I</td>
<td>(Evans-Pritchard, 1972, p. 230-231)</td>
</tr>
<tr>
<td>Bagli no khotno (The nest of the crane)</td>
<td>Gujarat, India</td>
<td>I</td>
<td>(Hornell, 1932, p. 156-157)</td>
</tr>
</tbody>
</table>

7 Annex II: Double-Sided lozenge String Figures

7.1 Kapiwa from the Trobriand Islands, Papua New Guinea

Step 1: Opening A

Step 2: Distally, insert 2345 into 1 loops. 2345 grasp 1f, 2 loops and 5 loops. Pass 1f to the dorsal side of the hands, while releasing 1 ...  

Step 2 continued: ...then, place hands facing each other.

Step 3: 1 picks up lower 2n in order to transfer the former 1 loops to the wrist.
Step 4: Pass 1 proximal to all intermediate strings. Proximally, insert 1 into 5 loop. Pick up 5f and return to position.

Step 4 continued: Release 5.

Step 5: Transfer 2 loops to 5.

Step 6: Proximally, insert 1 into 5 loop. 1 picks up 5n and returns to position.

Step 7: 2 picks up proximal 1f. Then 1 presses against the side of 2 to trap the string that runs from 1 to 2 and the string that runs from 1 to 5; wrists begin to turn toward the center.

Step 8: Release 5.

Step 8 continued: Release 1 while rotating the hands, turning the palms away.

Final figure of Kapiwa

7.2 Jasytata from Santa Teresita, Chaco, Paraguay
Step 1: Opening A

Step 2: Distally insert L2 into R2 loop. Pass L2 away from you distal to 5 loop, then towards you proximal to 5 loop. Pass L2 towards you distal to 1 loop, then pick up both R1f and R1n. L2 return to position.

Step 2 continued: Seize both R1n and R1f between L2 and L3. Release R1.

Step 2 continued: Distally, insert R1 into the loop seized between L2 and L3. Transfer this loop to R1.

Step 2 continued: Extend.

Step 3: Repeat Step 2 on the left hand by exchanging the role of L and R ...

Step 4: Release 2.

Final figure of Jasytata
### Annex III: Storer’s Systemology

<table>
<thead>
<tr>
<th>Objects</th>
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<tbody>
<tr>
<td><strong>Loops</strong></td>
</tr>
<tr>
<td>$\infty$</td>
</tr>
<tr>
<td>$Li\infty$</td>
</tr>
<tr>
<td>$Ri\infty$</td>
</tr>
<tr>
<td>$i\infty$</td>
</tr>
<tr>
<td>$W\infty$</td>
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<tr>
<td><strong>Strings</strong></td>
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<tr>
<td>$Lif$</td>
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<tr>
<td>$Rin$</td>
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<td>$if$</td>
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<td>$in$</td>
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<tr>
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<tbody>
<tr>
<td><strong>Openings</strong></td>
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<tr>
<td>$O$</td>
</tr>
<tr>
<td>$O.A$</td>
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<tr>
<td>$O.N$</td>
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<table>
<thead>
<tr>
<th>Operations on loops</th>
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</thead>
<tbody>
<tr>
<td><strong>Releasing</strong></td>
</tr>
<tr>
<td>$\Box$</td>
</tr>
<tr>
<td>$\Box$</td>
</tr>
<tr>
<td>$\Box R2\infty$</td>
</tr>
<tr>
<td>$\Box 2\infty$ or $\Box 2$</td>
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<tr>
<td>...</td>
</tr>
<tr>
<td><strong>Passing (over/under)</strong></td>
</tr>
<tr>
<td>$\rightarrow 1\infty$ (3$\infty$)</td>
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<tr>
<td>$\rightarrow 5\infty$ (2$\infty$)</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td><strong>Transferring</strong></td>
</tr>
<tr>
<td>------------------</td>
</tr>
<tr>
<td>( \rightarrow )</td>
</tr>
<tr>
<td>( \overrightarrow{1\infty} \to 3 )</td>
</tr>
<tr>
<td>( 5\infty \to 1 )</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td><strong>Rotations</strong></td>
</tr>
<tr>
<td>( &gt; )</td>
</tr>
<tr>
<td>( &lt; )</td>
</tr>
<tr>
<td>...</td>
</tr>
<tr>
<td><strong>Inserting</strong></td>
</tr>
<tr>
<td>( \overleftarrow{1\infty} \downarrow (5\infty) )</td>
</tr>
<tr>
<td>( 5\infty \uparrow (2\infty) )</td>
</tr>
</tbody>
</table>