Native American Dice Games and Discrete Probability

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Abstract.

Dice games were a common feature of Native American culture at the turn of the century. This essay reports on my examination of 40 Native American dice games described by James Culin (1992) in his two-volume 1907 work on Native American games from the point of view of discrete probability theory. I provide some initial answers to the question of whether or not the point values assigned to the outcomes of these games reflect the *a priori* probabilities of the outcomes.

Introduction.

In her book *Ethnomathematics,* Marcia Ascher (1991) analyzed a Cayuga dice game from a probabilistic point of view. The dice for the game were six peach stones each one painted black on one side. The dice were tossed in a wooden dish and points were assigned based upon the number of painted faces that landed up. The first player to achieve a pre-determined number of points was the winner. Ascher shows that the point values assigned to the outcomes of particular outcomes closely corresponded to the discrete probabilities of those outcomes. She writes, “The close correspondence of the point values used by the Cayuga and those based upon our ideas of probability strongly implies a probabilistic basis for the Cayuga’s choices. Were they based on some other unrelated criteria, we would not expect such a close match.” (p.90)

Clearly, varying the number of points awarded for different outcomes will make a game more interesting. In addition, one may speculate that a rarer outcome should be awarded more points that a common outcome. Ascher's remark is intriguing. Do the point values assigned to indigenous games of chance reflect the *a priori* probabilities of the outcomes?

This essay is a step towards an answer to this question. (As will be seen, additional questions concerning the relationship between mathematics and culture arise from the investigation.) I have limited my investigation at this juncture to the indigenous dice games of North America. My data are from Stewart Culin's massive collection of North American Indian games cataloged in his 1907 classic *Games of the North American Indian* (Culin, 1992).

***Games of the North American Indian*.**

Stewart Culin was curator of Ethnology at the Institute of Arts and Sciences of the Brooklyn Museum in New York City and was in charge of the exhibit of games of the world for the World Columbian Exposition in Chicago in 1893. His *Games of the North American Indian* was published in 1907 as the *24th Annual Report of the American Bureau of Ethnology*. For his report, Culin gathered examples of North American Indian games from a variety of sources including George Dorsey's collection at the Field Museum in Chicago and his own collecting trips to Indian reservations in 1901-1905 (Culin, 1992). He also compiled references to North American Indian games from a wide range of travel and anthropological works dating back to the late 18th century. All page numbers cited henceforth are from Culin's (1992) book.

The result of Culin's work is nothing short of astounding. Two volumes, comprising over 800 pages, of sketches, descriptions, and eye-witness reports of all sorts of games of chance and skill. The first volume, on games of chance, documents 130 cases of dice games alone. It is the dice games that are the focus of this study. Culin gives detailed descriptions of the dice as artifacts (including sketches), but infrequently provides details of play or the points assigned to the outcomes of dice tosses. As a result, I was able to glean clear data for only 40 games. Fortunately, the geographic and cultural spread of these 40 samples is quite broad and includes plains, west coast, eastern, and southwestern Indian groups. Here is an extended quotation from Culin (1992) that gives the flavor of his book and the data he provides.

"KERES. Acoma, New Mexico. (Cat. no. 4972, Brooklyn Institute Museum.)

Set of three stick dice (figure 124), 5 ½ inches in length, black on one side and plain white on the other. They were made for the writer by James H. Miller. He gave the name as owasakut. The counts are as follows: Three black counts 10; three white, 5; two white, 2; one white, 3. The game is counted around a circle of thirty stones, yow-wu-ni [figure 125], with little sticks called horses. There are three openings in the stone circle, which are called tsi-a-ma, door." (p.120)

(Note: This game is #15 of my study. See the table in the Appendix for a list and numbering of all games studied.)

North American Indian Dice Games

The dice in my sample were made from a wide range of material. There are bone disks, peach stones, plum stones, deer horn disks, wooden sticks, wood blocks, beaver teeth, walnut shells, acorns and corn kernels. Most of the dice are two-sided. In all of these dice one side that is painted black or blue or red or marked in some distinctive way and the other side is left plain or painted white. This makes the outcome of a toss of the dice easy to identify. Points are awarded based upon the nature of the sides of the dice that land upright. The wooden sticks are generally rounded on one side and flat on the other. Nevertheless, the sticks are still painted or marked to distinguish sides.

The play of the game is remarkably similar across cultures. The dice are tossed and points are awarded based upon the outcome. Total points are kept track of with counters or by collecting valuable objects from the opposing players. In some cases, the players move tokens around a circular playing diagram much like modern games Sorry or Parcheesi.

Although Culin (1992) provides considerable ethnographic information on the games, it is often spotty or incomplete. Here I focus only upon the relationship between the points awarded for an outcome and the a priori probability of that outcome.

However, before I get into that discussion, I must provide a caveat. I have not examined any of the actual dice described by Culin, so I have no assurances that the two or three outcomes for each die are equally likely. Indeed, it is quite likely that the dice are not perfectly weighted and that there are some physical biases to the construction of the dice that would, in effect, “load” them towards certain outcomes.

I do have a set of plum stone dice that were purchased in a gift shop in Mandan, North Dakota. Each plum stone is plain on one side and marked in the other. I tested each die and found that, after 100 tosses the a posteriori probability of the marked side landing upright was between 0.4 and 0.6. In the absence of similar data for the dice described by Culin, I have assumed equally likely outcomes in my analysis. For example, in the case of the Keres game cited above, I assigned probabilities as follows: Three black (1/8), three white (1/8), two white (3/8), one white (3/8).

Rarer Outcomes Earn More Points.

My initial question was whether or not the rarer (in terms of probability) outcomes earned the greater number of points. In 38 of the 40 dice games examined, the outcome with the highest point value also either had the lowest probability of occurrence of other point-earning outcomes or tied with other outcomes for the lowest probability of occurrence. The two exceptions (both from British Columbia) were a Nootka (my #34) and Thompson dice game (#25). In these games, the *most likely* of the point-earning outcomes earned the most points.

Thus, most of the games in my sample (95%) awarded the maximum number of points to at least one event with the minimum probability. Yet, in a majority of the games (24 out of 40), we have the peculiar result that outcomes bearing the same a priori probability are awarded differing point values (#1-4,7-11,15,16,24,26-29,31-33,36-39). An Apache game (#9) is representative of this phenomenon. The Apache game is played with three wooden sticks. Each stick is 10.75 inches long, flat on one side and rounded on the other side. The point counts were as follows:

All round sides up (probability 1/8)……….…….. 10 points

All flat sides up (1/8)…………………… … ………5 points

Two round, one flat (3/8)…………………..……..…2 points

Two flat, one round (3/8)…………………..……...…3 points

Roundedness appears to be more highly valued than flatness in the game, but rather inconsistently. Three rounds are better than three flats, but two flats are marginally better than two rounds.

In only 14 games (#5, 6, 12-14, 17-23, 35, 40) can we confirm that the point value for the rarest outcome is the highest point value of any outcome. A Menominee dice game (#6) is representative of this group.   The Menominee game is played with eight plum stone dice. On side of each stone is painted white and the other side is painted red. The point counts were as follows:

All red sides up (probability 1/256) …………………..……40 points

Seven red and one white (8/256) ………..……………….20 points

Six red and two white (28/256)…………….…………4 points

Five red and three white (56/256)…………. …….…….. 1 point

All other outcomes have zero point value.

This Menominee game is also a nice example of the games in which the most likely scoring outcome earns the fewest number of points. Twenty-nine of the 40 games (# 1, 2, 4, 6, 8-24, 26, 29, 32, 33, 35, 36, 38, 39) had this feature.

We do observe that there is some correspondence between probabilities and points. The highest number of points is usually associated with the lowest probability and the lowest number of points is usually associated with the highest probability. Games designed with the mathematics in mind would have this feature.

More Points Doesn't Mean Rarer Outcome.

As shown in the previous section, the highest scoring outcomes in Native American dice games usually have the minimum probability of occurrence. Yet, in many cases, there are lower scoring outcomes that also bear the minimum probability. One may well ask about the nature of these lowest probability – lower point outcomes.

Twenty-one of the twenty-four games in question easily divide themselves into two categories. The largest category (15 games) was comprised of games in which the dice had two simple distinguishable sides: marked/unmarked, black/white, red/black or round/flat. For the games in this category, there was a specific preferred side. When all dice came up with that side, the maximum number of points was awarded. Equally likely, but less preferred, outcomes, like the case when all dice come up with the less preferred side, earned fewer points. The Apache game (#9) described above is an example of the games in this category. The preferred side in these games is a purely cultural choice and has nothing at all to do with the mathematics. It is worthy of note however, that "all blank" and "all flat" are never the preferred outcomes.

The second category consists of games for which one die has a distinguishing unique mark (#4, 8, 24, 26, 31, and 32). For example, one of the dice may be notched (#31) or one may be banded (#4). In these games, the highest points are awarded for tosses in which the special die lands differently from the others. That is, the special die is up (however defined) and the remaining dice are down, and vice versa. The all-marked and all-plain outcomes score lower in these games even though their probabilities match that of the special die outcomes. A Grosventres game (#4) is paradigmatic of this category. The Grosventres game uses four 10-inch long wooden sticks, flat on both sides. Two of the sticks are painted green with one side of each marked in red. The other two sticks are painted red with one side of each marked in green. One of the red sticks is banded. The point counts for unique scoring outcomes were as follows:

All marked sides up (probability 1/16) ……………………4 points

All plain sides up (1/16) …………………………………...4 points

Banded stick has plain side up and

the others have marked side up (1/16)………………..…….6 points

Banded stick has marked side up and

the others have plain side up (1/16)………………..…….6 points

In this case, the banded stick has prominence in the game.

The remaining three games are quite unlike the bulk of the sample in either the construction of the dice or the method of scoring.   Two of the games (a Haida game (#28) and a Kodiak Island game (#11)) have a single die that is irregular in shape. Points are awarded based upon the orientation of the die when it comes to rest. I doubt that the various positions are equally likely, but I did treat them as such in this study. I can offer little in the way of analysis for these games without access to the dice themselves.   The other game is a Dakota plum stone dice game (#27). Each plum stone has an animal (spider, lizard or turtle) engraved on one side. Each animal appears on two stones. The other side of each stone was blank. Points were awarded based upon the number and type of animal that came up. Thus, the maximum point score resulted from all six animals showing up (probability 1/64). Fewer points would be earned from equally likely lesser quantities of animals (for example, two turtles and four blanks).

The data suggest that when outcomes with equally low probabilities are awarded different point values in North American Indian dice games there is some cultural preference for one of the outcomes. Either there is a specially marked die or, most frequently, there is a preferred color or pattern on the dice. I think this preference for a particular color or pattern is what is in play in the case of the anomalous Nootka and Thompson dice games mentioned earlier.

The Thompson game (#25) was played with four beaver teeth dice. Each tooth was marked on one side and left plain on the other. The markings distinguished the dice from one another. The "man" tooth had eight transverse lines marked in it and its "mate" had five transverse lines each with a dot in the middle. The other two dice were called "mates" as well and marked alike with triangles. After the dice were tossed, points were awarded based upon the result of the toss. The point counts were as follows:

All blank sides up (probability 1/16)………..……..2 points

All marked sides up (1/16)…………… … ………2 points

A triangular face up and the rest blank (2/16)……14 points

Dotted tooth up and the rest blank (1/16)…….……8 points

The "man" up and the rest blank (1/16) ……………4 points

Curiously, the most likely of the point-earning outcomes has the highest point value.

The Nootka game (#34) is also played with four dice. These dice are made of bone, but cut in the form of beaver teeth. Each die is marked on one side and left blank on the other. Two of the dice are marked with spots and the other two with diagonal lines. The point counts were as follows:

All marked sides up (probability 1/16) ……………………2 points

All blank sides up (1/16) …………………………………2 points

Two spotted dice up and the others blank (1/16)…………1 point

Two lined dice up and the others blank (1/16) …….…….. 2 points

Two lined up, one spotted up and one blank (2/16)...……..4 points

Once again, the most probable of the point-earning outcomes gets the most points.

In these games it seems that a preferred side (triangles) or a preferred combination (two-lined, one-spotted) trumps the natural probabilities. These preferences may be explainable in terms of myth or some other cultural component, but that's a topic for later investigation.

Conclusion and New Questions.

The focus of this study was the relationship between the point values awarded to specific outcomes in Native American dice games and the *a priori* probabilities of these outcomes. The data suggest that the games do award more points to less probable outcomes. However, there are at least two interesting questions that emerged from the study that this paper must leave unanswered.

First, the data hint at an interesting and complicated interweaving of the mathematics (as expressed by point values) and other cultural concerns (as expressed by preferred outcomes). This interweaving is apparent in the Nootka game described above. Another example that hints at cultural preference trumping mathematical probability is a Cheyenne dice game (#2). In this game, five two-sided dice are tossed. All five dice are blank on one side. Three dice have crosses on the other side, while two have bears drawn on the other side. The outcomes "two blank-three crosses" and "two bears-three crosses" have identical *a priori* probabilities (1/32). However, the outcome with two bears is worth 8 points while the outcome with two blanks is worth 3 points. There seems to be a cultural influence woven into the game that makes the two bears a more valuable outcome than the two blanks. The details of this weaving remain to be discovered.

The second interesting question that arises from this study is *how* the specific point values came to be assigned to the outcomes of the various Native American dice games. Were the points assigned based upon *a priori* probabilities or did they evolve over time based upon empirical evidence of outcome frequencies? I think it very unlikely that we may ever be able to discern whether or not the point values for the games were based upon *a priori* probabilities. To my knowledge, nothing in the ethnographical or archeological record suggests that this is the case. Indeed, it seems more likely that the point distributions evolved over time as a result of *a posteriori* analysis of outcomes. It isn't difficult to imagine dice game players modifying the point values of outcomes to make the game more exciting. However, it seems that the published record is mute on this question.

Thus, we cannot say whether the correspondences between point values and probabilities were by conscious design or the result of experience in playing the games. Nor do we understand clearly how the cultural values of the dice game players influenced the choice of point values for outcomes. However, we can affirm that Native American dice games do reflect discrete probabilities in their design.

Appendix: Table of Games Studied

The 40 games discussed in this paper are all from Culin (1992). Here is the list in order of appearance in Culin's book. Each is followed by the number of the page on which it appears. The cultural nomenclature is Culin's and does not necessarily reflect current terminology. The game numbers are peculiar to this study.

Game Number	Culture (Culin, 1992, page)	Game Number	Culture (Culin, 1992, page)
1	Arapaho (53)	21	Miwok (144)
2	Cheyenne (58)	22	Choctaw (146)
3	Cree (69)	23	Nishinam (155)
4	Grosventres (70)	24	Songish (157)
5	Grosventres (72)	25	Thompson (157)
6	Menominee (75)	26	Twana (158)
7	Micmac (77)	27	Dakota (179)
8	Sauk & Foxes(85)	28	Haida (189)
9	Apache (87)	29	Tewa (190)
10	Navaho (95)	30	Tewa (192)
11	Kodiak Is. (104)	31	Tewa (194)
12	Onondaga (112)	32	Tigua (195)
13	Onondaga (112)	33	Makah (197)
14	Seneca (113)	34	Nootka (198)
15	Keres (120)	35	Washo (199)
16	Keres (121)	36	Havasupai (200)
17	Klamath (137)	37	Maricopa (201)
18	Tejon (138)	38	Walapai (207)
19	Wiktchamne (139)	39	Zuni (221)
20	Awani (143)	40	Zuni (223)

References

Ascher, M. (1991). *Ethnomathematics.* Pacific Grove: Brooks/Cole.

Culin, S. (1992). *Games of the North American Indians, Volume 1, Games of Chance.* Lincoln: University of Nebraska Press.